Some Calculations for the GS-13 Readout and Performance

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1 Summary

In this document:

1. We derive a model for the combined electrical and mechanical response for the GS–13 with the –03 readout.

2. We derive a model of the noise of the –03 readout, and calculate the noise floor for that readout.

3. We show that the noise floor of the present readout is limited by the current noise of the LT1007.

4. We calculate that we can build a readout with a noise floor of 1.3e-11 m/$\sqrt{\text{Hz}}$ at 1 Hz and 1.8e-13 m/$\sqrt{\text{Hz}}$ at 10 Hz.

5. We note that the noise floor calculated by Rodgers '92 seems a bit optimistic.

2 Response of the GS–13

2.1 Mechanical Response

We model the seismometer as a mass, m, on a spring, k, with damping b. The mass location is x_m, the base of the spring is attached to the ground (which we are trying to measure) at location x_g.

$$m\,\ddot{x}_m = -k(x_m - x_g) - b(\dot{x}_m - \dot{x}_g) + I \cdot G \tag{1}$$

where I is the current flowing in the geophone coil, and G is the generator constant. It is more convenient to consider the differential motion between the ground and the mass, so we let $x = x_m - x_g$, and equation 1 becomes

$$\ddot{x} = -\frac{k}{m}\,x - \frac{b}{m}\dot{x} + \frac{I \cdot G}{m} - \ddot{x}_g. \tag{2}$$

We model the electrical circuit as a voltage generator (the moving geophone) driving a series combination of the 9 kΩ coil resistance R_c, the ??? H coil inductance L_c, and the load of the readout Z_r. We define the total impedance Z_t as $Z_t = R_c + s\,L_c + Z_r$. The moving mass generates a voltage

$$V = -\dot{x} \cdot G, \tag{3}$$

So the current generated will be

$$I = -\frac{\dot{x} \cdot G}{Z_t}. \tag{4}$$

substituting this expression for the coil current into the dynamics equation 1 yields

$$\ddot{x} = -\frac{k}{m}\,x - \frac{b}{m}\dot{x} - \frac{G^2}{m\,Z_t}\dot{x} - \ddot{x}_g. \tag{5}$$

The simplest readout scheme for a geophone is to put the coil into the positive terminal of a low–noise op–amp. In this case, Z_t is effectively infinite, and the readout doesn't change the dynamics of the instrument at all. However, the accelerometer readout used by the GS–13 employs a "negative resistor" to make the damping very high.

2.2 The Readout Circuit

2.2.1 The negative resistor

The present pre–amp circuit is not shown in any figure. Z_1 is the impedance from the op–amp output to the negative input, r_2 is the resistor from the output to the positive input, and r_3 is the resistor from the positive input to ground. Assuming the configuration is stable, so that the usual op–amp simplifications apply, $V_+ = V_-$ and

$$V_+ = V_{out} * r_3/(r_2 + r_3). \tag{6}$$

Therefore, if one applies a voltage V_- to the negative input, the current, i_r, flowing into the readout circuit, and then through Z_1 will be

$$i_r = \frac{V_- - V_{out}}{Z_1}. \tag{7}$$

$$i_r \cdot Z_1 = V_- - \frac{r_2 + r_3}{r_3} V_- \tag{8}$$

$$i_r \cdot Z_1 = -\frac{r_2}{r_3} V_-, \tag{9}$$

$$i_r \cdot -\frac{r_3}{r_2} Z_1 = V_-, \tag{10}$$

so we can model the op–amp as a negative impedance, Z_n, of

$$Z_n \equiv -\frac{r_3}{r_2} Z_1 \tag{11}$$

2.2.2 The readout, in general

We want to relate the voltage out of the op–amp to the motion of the ground. For this, we need the behavior of the readout circuit and the behavior of the mechanics. The current through the readout impedance, i_r flows from the negative terminal of the op–amp to the op-amp output. Therefore

$$i_r = \frac{V_- - V_{out}}{Z_1} \tag{12}$$

since $V_- = V_{out} r_3/(r_2 + r_3)$,

$$i_r = \frac{V_{out}}{Z_1} \cdot \frac{-r_2}{r_2 + r_3}. \tag{13}$$

The current into the readout circuit is equal to the current generated by the coil, which can be calculated from equation 3.

$$i_r = \frac{\dot{x}\, G}{Z_t}, \text{ therefore} \tag{14}$$

$$\frac{V_{out}}{Z_1} \cdot \frac{-r_2}{r_2 + r_3} = \frac{\dot{x}\, G}{Z_t}, \text{ so} \tag{15}$$

$$V_{out} = \frac{\dot{x}\, G\, Z_1}{Z_t} \cdot \big(- \frac{r_2 + r_3}{r_2} \big) \tag{16}$$

2.2.3 The impedance of the GS–13 circuit

The GS–13 readout uses a parallel combination of r_1 and C_1 to form the impedance Z_1. The effective impedance of the readout is, therefore

$$Z_r = -\frac{r_3}{r_2} \cdot \frac{\frac{r_1}{s\, C_1}}{r_1 + \frac{1}{s\, C_1}} \tag{17}$$

$$Z_r = -\frac{r_3}{r_2} \cdot \frac{r_1}{1 + s\, C_1\, r_1} \tag{18}$$

to simplify the expression, we define

$$R_r \equiv \frac{r_3}{r_2} \cdot r_1, \tag{19}$$

and

$$C_r \equiv \frac{r_2}{r_3} \cdot C_1, \tag{20}$$

so Z_r becomes

$$Z_r = -\frac{R_r}{1 + s\, C_r\, R_r}. \tag{21}$$

Note that the negative sign is treated explicitly, so R_r and C_r are positive, and the time constant $R_r\,C_r$ is the same as $r_1\,C_1$. With this, we calculate Z_t, the total impedance seen by the GS–13 as

$$Z_t = R_c + s\,L_c + Z_r = R_c + s\,L_c - \frac{R_r}{1 + s\,C_r\,R_r}. \tag{22}$$

a bit of algebra yields

$$Z_t = \frac{(R_c - R_r) + s\,(L_c + R_c\,R_r\,C_r) + s^2\,L_c\,R_r\,C_r}{1 + s\,C_r\,R_r}. \tag{23}$$

We define the equivalent resistance of the load to be $R_e = R_c - R_r$,

$$Z_t = \frac{R_e \cdot (1 + s\,(\frac{L_c}{R_e} + \frac{R_c\,R_r\,C_r}{R_e}) + s^2\,\frac{L_c\,R_r\,C_r}{R_e})}{1 + s\,C_r\,R_r}. \tag{24}$$

We will now try to get rid of some of these terms which don't matter below 100 Hz. The values of the components are given in table 1. The value of the coil given in table 1 assumes

Part	witness value	Schematic value
r_1	15.8 kΩ	4.87 kΩ
r_2	2.59 kΩ	5.98 kΩ
r_3	1.00 kΩ	8.87 kΩ
C_1		47 nF
R_n	6.10 kΩ	
C_n		122 nF
R_c	9.1 kΩ	
L_c	207 H ??	
R_e	3.0 kΩ	
mass		5 kg
f_0		1 Hz
b	70 N*s/m ??	

Table 1: Values of selected components for GS-13 #577 (an ETF witness)

that the L/R time constant of the coil is 7 Hz. I have no data to back this up, but it gives a reasonable transfer function. To simplify equation 24, we note that for the denominator, $1/2\pi R_r C_r \approx 215$ Hz and will be ignored. In the numerator, $R_c\,R_r\,C_r = 6.7 \ll L_c$. We can simplify the total impedance to

$$Z_t \approx R_e \cdot \big(1 + s\frac{L_c}{R_e} + s^2\frac{L_c\,R_r\,C_r}{R_e}\big). \tag{25}$$

The s^2 term has been improperly ignored, since

$$\sqrt{\frac{R_e}{L_c\, R_r\, C_r}}/2\pi \approx 22 \text{ Hz}. \tag{26}$$

We use

$$Z_t \approx R_e\big(1 + s\frac{L_c}{R_e}\big). \tag{27}$$

2.3 Response of the GS–13

2.3.1 Mechanical response, including the readout circuit

With this expression for the load seen by the geophone, we can look again at the dynamics in equation 5, and take it's Laplace transform.

$$\ddot{x} = -\frac{k}{m}\,x - \frac{b}{m}\dot{x} - \frac{G^2}{m\,Z_t}\dot{x} - \ddot{x}_g.$$

$$s^2\,x = -\frac{k}{m}\,x - \frac{b}{m}\,s\,x - \frac{G^2\,s}{m\,Z_t}x - s^2\,x_g. \tag{28}$$

$$x\Big(s^2 + \frac{b}{m}\,s + \frac{k}{m} + \frac{G^2\,s}{m\,Z_t}\Big) = -s^2\,x_g. \tag{29}$$

We substitute our expression for Z_t and get

$$x\left(\big(s^2 + \frac{b}{m}\,s + \frac{k}{m}\big)\big(1 + \frac{L_c}{R_e}\,s\big) + \frac{G^2\,s}{m\,R_e}\right) = -s^2\,x_g\,\big(1 + \frac{L_c}{R_e}\,s\big) \tag{30}$$

which can be expanded to

$$x\Big(s^3\frac{L_c}{R_e} + s^2(1 + \frac{b}{m}\,\frac{L_c}{R_e}) + s\,(\frac{b}{m} + \frac{k}{m}\,\frac{L_c}{R_e} + \frac{G^2}{m\,R_e}) + \frac{k}{m}\Big) = -s^2\,x_g\,\big(1 + \frac{L_c}{R_e}\,s\big) \tag{31}$$

We can now write the transfer function from ground motion to relative motion in the geophone as

$$\frac{x}{x_g} = \frac{-s^2\big(1 + \frac{L_c}{R_e}\,s\big)}{\Big(s^3\frac{L_c}{R_e} + s^2(1 + \frac{b}{m}\,\frac{L_c}{R_e}) + s\,(\frac{b}{m} + \frac{k}{m}\,\frac{L_c}{R_e} + \frac{G^2}{m\,R_e}) + \frac{k}{m}\Big)}. \tag{32}$$

It's instructive to consider the case with no inductance (which is what they do in the manual). In that case, the transfer function simplifies to

$$\frac{x}{x_g}\text{ (no inductance)} = \frac{-s^2}{s^2 + s\,(\frac{b}{m} + \frac{G^2}{m\,R_e}) + \frac{k}{m}}. \tag{33}$$

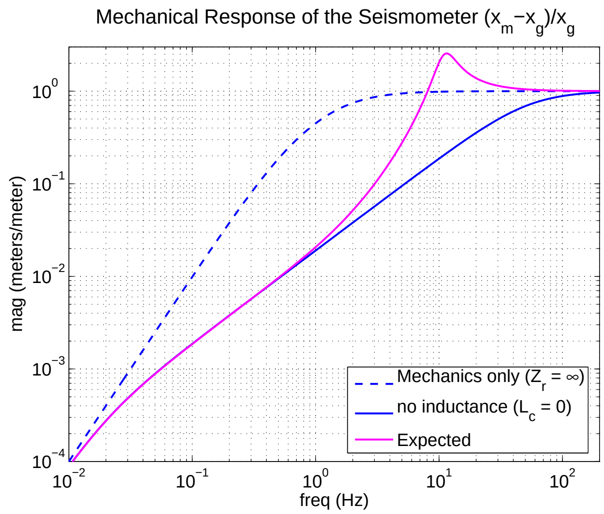

Figure 1: Transfer function of the differential motion between ground and test mass as a function of ground motion.

2.3.2 Total response to ground motion

Now that we have calculated the mechanical response of the system, we can use equation 16 to calculate the response of the instrumented geophone to ground acceleration.

$$\frac{V_{out}}{s^2\,x_g} = \frac{s\,x}{s^2\,x_g}\frac{G\,Z_1}{Z_t}\cdot\big(-\frac{r_2+r_3}{r_2}\big) \tag{34}$$

We let $Z_t = R_e(1 + s\,L_c/R_e)$ and this becomes

$$\frac{V_{out}}{s^2\,x_g} = \frac{-s}{\left(s^3\frac{L_c}{R_e} + s^2(1+\frac{b}{m}\frac{L_c}{R_e}) + s\,(\frac{b}{m}+\frac{k}{m}\frac{L_c}{R_e}+\frac{G^2}{m\,R_e}) + \frac{k}{m}\right)}\left(\frac{G\,r_1}{R_e}\right)\left(-\frac{r_2+r_3}{r_2}\right) \tag{35}$$

This response is plotted in figure 2.

The manual claims that the gain of the –03 model of the GS–13 is 500 V/(m/s^2) at 1 Hz. At 1 Hz, our model gives a response of 47 V/(m/s^2) from the pre–amp, and the witness GS–13s have an additional differential gain of 10 (+5 and -5), giving a total calculated sensitivity of 470 V/(m/s^2).

However, the manual seems to completely ignore the impact of the coil inductance, and claims the dynamics are one zero at DC, and real poles at .125 rad/sec and 314 rad/sec (20 mHz and 50 Hz). The measured transfer function looks much more like our model.

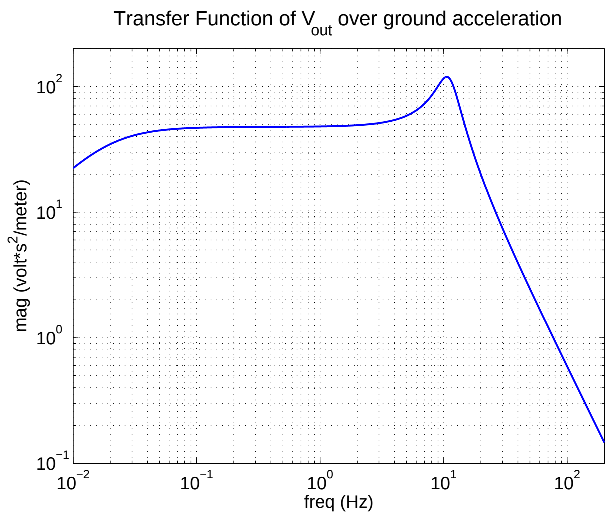

Figure 2: Transfer function of the pre–amp output voltage/ ground acceleration

3 Noise

3.1 Impedance of the Geophone

The geophone presents an impedance to the op–amp which is a function of the dynamics of the instrument, and the static impedance of the coil resistance and inductance.

We recall the two basic equations for the geophone dynamics, and, if we ignore the ground motion, we get

$$s^2 x = -\frac{k}{m} x - \frac{b}{m} s\, x + I \cdot G, \text{ and,} \tag{36}$$

$$V = s\, x\, G \tag{37}$$

We do some algebra

$$x\Big(s^2 + + \frac{b}{m} s + \frac{k}{m}\Big) = I \cdot G, \text{ replace } x \text{ with } V, \tag{38}$$

$$\frac{V}{s\, G}\Big(s^2 + + \frac{b}{m} s + \frac{k}{m}\Big) = I \cdot G, \text{ and see that,} \tag{39}$$

$$V = I \frac{G^2\, s}{\left(s^2 + \frac{b}{m} s + \frac{k}{m}\right)} \equiv I \cdot Z_{mm} \tag{40}$$

We define Z_{mm} as the impedance presented by the moving mass. We see that the impedance is peaked at the natural frequency of the resonance, and is proportional to the generator constant squared.

The total impedance of the geophone, z_g will therefore be

$$z_g = R_c + s\,L_c + Z_{mm} = R_c + s\,L_c + \frac{G^2\,s}{(s^2 + \frac{b}{m}s + \frac{k}{m})} \tag{41}$$

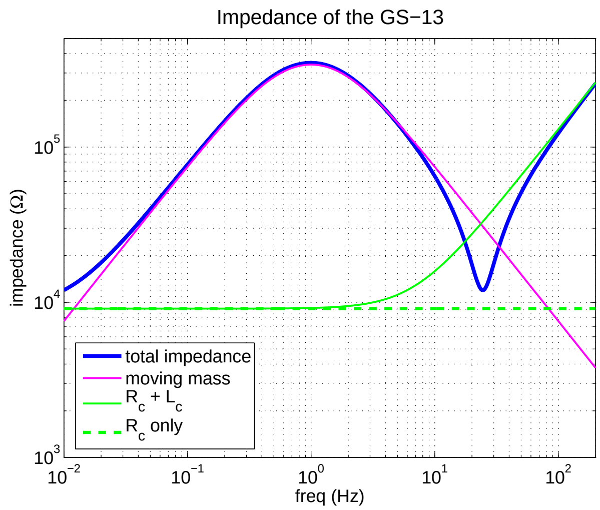

Figure 3: Impedance of the GS–13, including the impact of the moving mass

3.2 Amplifier Noise

We consider the circuit in figure 4, which is a model of the pre–amplifier which includes 3 noise sources: 1) $v_t g$, thermal noise from the geophone, 2) v_{t1} thermal (johnson) noise from readout resistor r_1, 3) v_n, amplifier voltage noise, and 4) i_n, amplifier current noise. I have not included the thermal noise from the resistors r_2 and r_3.

1) ***Thermal noise from the geophone***: The geophone is a source of thermal noise. This comes both from the Johnson noise of the coil resistance, and from the thermal noise of the suspension. The amount of voltage noise generated by the geophone is v_{tg}. We calculate the noise generated as

$$v_{tg} = \sqrt{4kT \cdot \mathrm{Real}(Z_g)} \tag{42}$$

where we assume the resistance is the real part of the impedance Z_g presented by the geophone (defined in equation 41. Then, we calculate the impact of the thermal noise of the geophone on the output of the pre–amp by comparing expressions for the voltage at V_-.

$$V_- = v_{tg} - i_1\,z_g = V_{out}\frac{r_3}{r_2 + r_3} \tag{43}$$

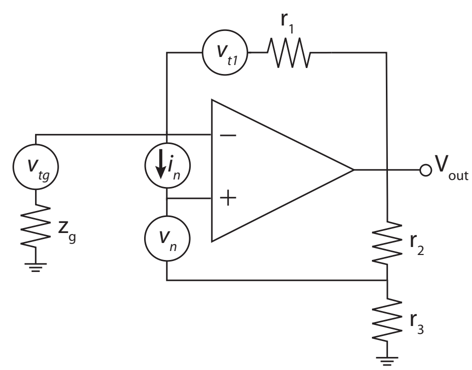

Figure 4: Noise model for the amplifier

$$i_1 = \frac{V_- - V_{out}}{r_1} = \frac{V_{out}}{r_1}\Big(\frac{-r_2}{r_2 + r_3}\Big) \tag{44}$$

The equation 43 becomes

$$v_{tg} - \frac{V_{out}}{r_1}\Big(\frac{-r_2}{r_2 + r_3}\Big)\, z_g = V_{out}\frac{r_3}{r_2 + r_3}, \text{ and so} \tag{45}$$

$$v_{tg} = V_{out}\Big(- \frac{z_g}{r_1}\frac{r_2}{r_2 + r_3} + \frac{r_3}{r_2 + r_3}\Big), \text{ therefore} \tag{46}$$

$$V_{out} = v_{tg} \cdot \frac{r_1 \cdot (r_2 + r_3)}{-z_g\, r_2 + r_1\, r_3} \tag{47}$$

2) ***Thermal noise from the readout resistor***: We also consider output voltage from the thermal noise voltage $v_{t1} = \sqrt{4kT\, r_1}$ generated by the resistor r_1. The output voltage is calculated by setting the current through z_g equal to the current through r_1.

$$i = \frac{0 - V_-}{z_g} = \frac{V_- + v_{t1} - V_{out}}{r_1} \tag{48}$$

$$v_{t1} = \frac{-r_1}{z_g} V_- - V_- + V_{out} \tag{49}$$

$$v_{t1} = V_{out} \cdot \Big(\frac{-r_1}{z_g}\frac{r_3}{r_2 + r_3} - \frac{r_3}{r_2 + r_3} + \frac{r_2 + r_3}{r_2 + r_3}\Big) \tag{50}$$

$$v_{t1} = V_{out} \cdot \Big(\frac{-r_1}{z_g}\frac{r_3}{r_2 + r_3} + \frac{r_2}{r_2 + r_3}\Big) \tag{51}$$

$$V_{out} = v_{t1} \cdot \frac{z_g \cdot (r_2 + r_3)}{-r_1\, r_3 + z_g\, r_2} \tag{52}$$

3) ***Amplifier Voltage Noise***: For an ideal op–amp, $V_- = V_+$, and so

$$V_{out} \cdot (\frac{z_g}{z_g + r_1}) = V_{out} \cdot (\frac{r_3}{r_3 + r_2}) + v_n, \text{ and so} \tag{53}$$

$$V_{out} = v_n \cdot \frac{1}{(\frac{z_g}{z_g+r_1} - \frac{r_3}{r_3+r_2})} \tag{54}$$

4) ***Amplifier Current Noise***: Again, we set $V_- = V_+$.

$$V_- = i_n(r_1 \parallel z_g) + V_{out}\frac{z_g}{z_g + r_1} \tag{55}$$

$$V_+ = -i_n(r_2 \parallel r_3) + V_{out}\frac{z_3}{z_3 + r_2} \tag{56}$$

and so

$$V_{out} = i_n \cdot \frac{1}{(\frac{z_g}{z_g+r_1} - \frac{r_3}{r_3+r_2})} \cdot (r_1 \parallel z_g + r_2 \parallel r_3) \tag{57}$$

These four noise sources are added in quadrature, which implies that we use only the absolute values.

3.3 Measured Noise

The measured noise of the GS–13 is 1e-5 dspace/rtHz according to ETF entry 247. The

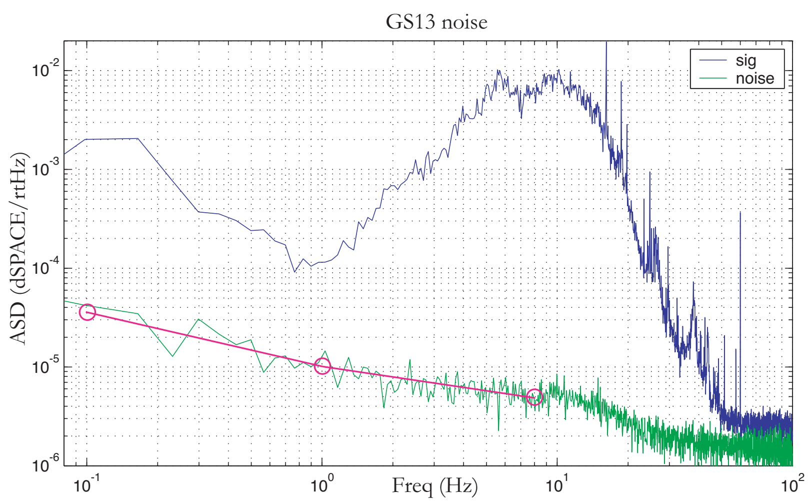

Figure 5: Measured noise of the GS–13

noise is changing as $1/f^{1/2}$ between about 0.1 Hz and 10 Hz. It should be sqrt(2) larger, from the double to single sided ASD error. The gain of the dSPACE is .1 dSPACE/volt, the gain of the differential readout (at the time) was 101, and the gain of the differential driver for the witness channels is 10. Therefore the noise at the output of the pre–amp at 1 Hz is

$$V_{out} = 1e - 5 \cdot \frac{\sqrt{2}}{.1 * 101 * 10} = 1.4e - 7 \frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}} \tag{58}$$

3.4 Comparing the prediction to a measurement

Using the relations developed in section 3.2, along with the published noise performance of the LT1007, we can predict the noise at the output of the GS–13 pre–amp. That noise is shown in figure 6, along with the simple fit to the measured noise shown in figure 5. We can see that the measured noise is close to the predicted noise, and that the predicted noise is completely dominated at 1 Hz by the current noise of the LT1007.

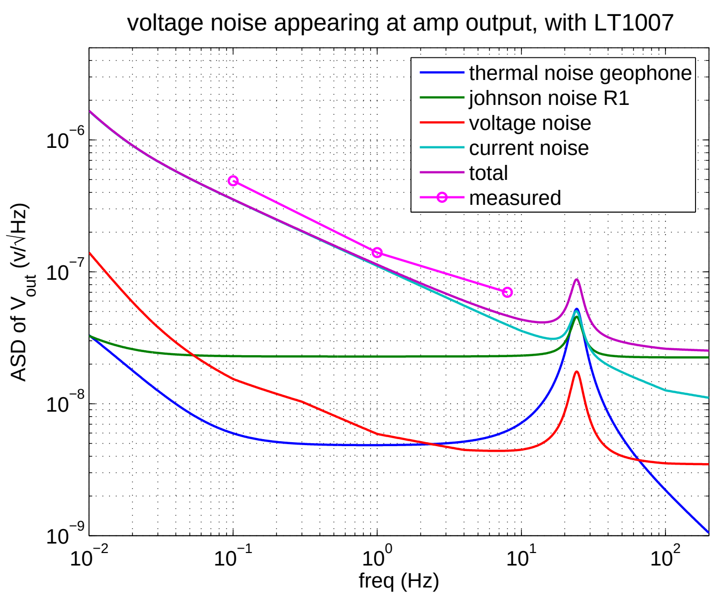

Figure 6: Calculated noise floor of the GS–13

The noise from the as–built –03 readout circuit is clearly not as good as it could be. There are two things it can be compared against. First is an identical readout with an amplifier which has a lower current noise, such as the LT1012. The second is a readout like that described by Rodgers, which is to hook one lead of the geophone to ground, and the other lead to the non–inverting input of an op–amp.

Knowing the noise of the instrument, one can then calculate the noise floor (m/$\sqrt{\mathrm{Hz}}$)

by dividing the noise floor (V/$\sqrt{\text{Hz}}$) by the instrument response (V/m). The calculated noise floors for several configurations are shown in figure 7.

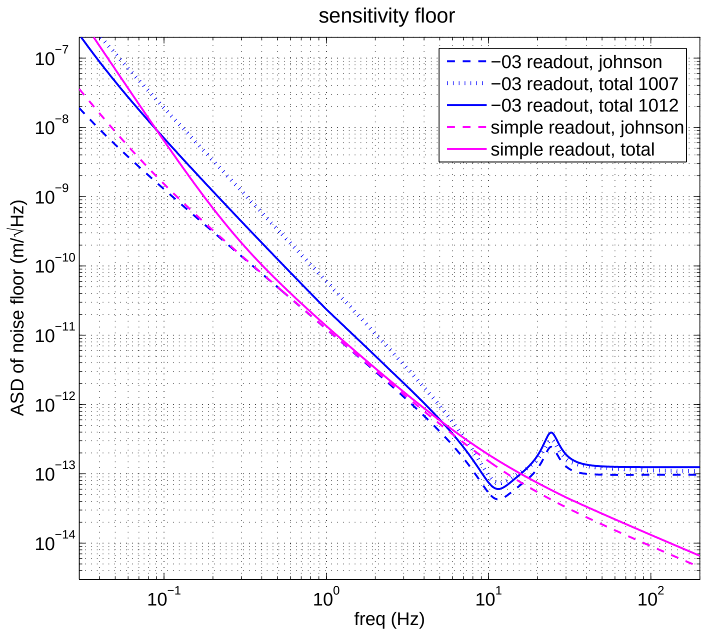

Figure 7: Calculated noise floor of the GS–13. The blue lines are for the –03 style readout, and the magenta lines are for the simple readout. The dashed lines are the noise floor assuming only the thermal noise components. These are quite similar. The solid lines use the LT1012 as the readout, and the dotted line is the as–built configuration with the the LT1007.

3.5 Noise for the simple readout

Figure 8 is a plot of the noise components for the simple readout. The LT1012 does a pretty good job, but the voltage noise at 10 Hz and above does degrade the performance. At 10 Hz, the total noise is about 20% larger than the thermal noise. One could look around for a better amp. For example, the LT1792 is a JFET input which is quieter, but it burns about 10x the power, which is bad for our application. A more thorough investigation of amplifiers would be useful.

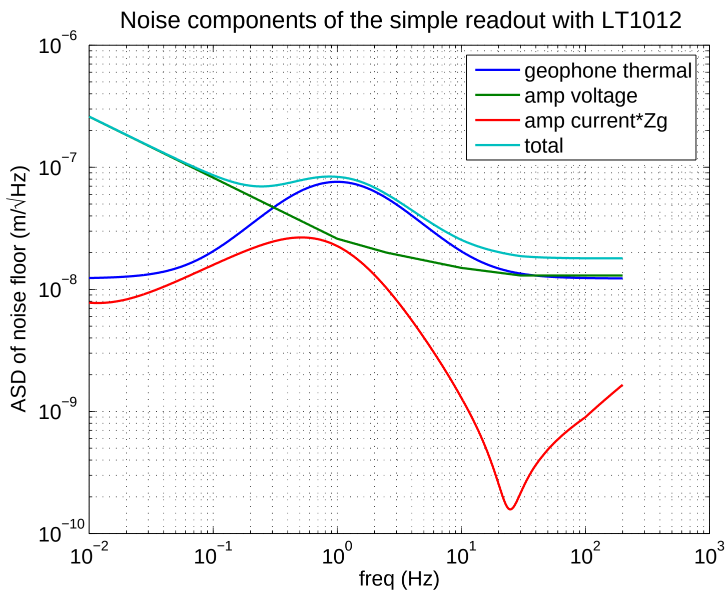

Figure 8: Noise contributions for the simple readout scheme with an LT1012