Design and operation of a very large ring laser gyroscope

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The design and initial operation of a vertical square He–Ne ring laser G0 with a perimeter of 14 m is discussed. This builds on earlier demonstrations of the feasibility of large ring lasers (perimeter ~4 m) for single-mode gyroscope operation and with lesser pulling than navigation gyroscopes. With servoing of the rf excitation to yield single-mode operation, G0 gave a quality factor 1×10^{12} and a Sagnac line with a frequency of 287.8 ± 1.0 Hz induced by Earth rotation Ω_E. This has confirmed some vital questions over the feasibility of very large gyroscopes for geodetic measurements at the level of $10^{-9}\ \Omega_E$. © 1999 Optical Society of America

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1. Introduction

Ring laser gyroscopes of area $A < 0.02\ m^2$ are commonly used for inertial navigation rotation sensors. Despite earlier doubts of the value of such dimensional increases, several greatly upscaled He–Ne ring lasers ($\lambda = 633$ nm) with areas $A \sim 1\ m^2$ (here called large) have been successfully built.[1] For example, a ring laser (called C-I) in the Cashmere cavern of the University of Canterbury has $A = 0.755\ m^2$. A second-generation monolithic square He–Ne ring laser C-II with $A = 1\ m^2$ was installed in 1997 at Cashmere. Initial results from C-II[2,3] give rotation sensitivities that are superior to those of atomic gyroscopes—which are themselves superior to other inertial sensors of rotation such as neutron interferometers and super-conducting quantum interference devices[1]—and also give Earth-induced Sagnac frequency stabilities of 100 parts per million (ppm) over a few days; this is further reduced to 25 ppm when postprocessing modeling of pressure and temperature effects is included. C-II was built in 1995–1996 by Carl Zeiss for the Bundesamt für Kartographie und Geodäsie, Frankfurt, and Forschungseinrichtung Satellitengeodäsie of the Technische Universität München. In summary, C-I proved that large rings are possible, and C-II proved that with optimal design high resolution can be achieved.

The potential advantages of dimensional scaling[1–4] include a reduction in frequency pulling, the relative effects on a simple model being proportional to the inverse cube of the ring laser perimeter.[1] The sevenfold linear scaling of dimension from the navigation gyroscopes to these large ring lasers thus builds in a considerable advantage with which to balance the increased problems with mechanical stability and with maintenance of single-mode operation. At this stage, the Sagnac frequency from C-II (nominally 79.40 Hz) has been stabilized to a few millihertz over a few days. Significant improvement is expected on this; the stability is improved by a factor of 100 when the time scale is reduced to a few hours. Because this permits resolution of the quantum shot-noise contribution to the linewidth, it is evident that we have reached the required design goal in principle and are limited only by environmental effects mediated by backscatter-induced drifts. This strengthens the motivation for considering ring laser designs with inherently less backscatter-induced drift, and so increasing the area and perimeter of the ring laser while improving the mechanical stability of the mirror separations, as well as improving the mirror quality.

In a further development, this collaboration has

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commenced design work on a He–Ne laser called G (for Grossring) with $A \sim 16$ m^2 (here called very large). G is of geodetic interest for measuring short-term fluctuations (with periods of hours to days) in the rate Ω_E of Earth rotation.[1] Short-term fluctuations in Earth rotation are known from very long baseline interferometry and laser ranging only to parts in 10^9 (Refs. 5–8). Although this further scaling of the ring laser dimensions continues to be advantageous and is essential for raising the stability of the gyroscope toward the level (parts in 10^9) required for such geodetic measurements, the necessary further upscaling of dimensions (say a factor of 16 in the area A) raises new questions of principle as well as cost, and full prior analyses and feasibility tests are vital for the G project and to assess the general potential of ring laser upscaling.

A primary requirement of any sensor of Earth rotation fluctuation is that instrumental drift in the measurement of rotation rate should be reducible to less than a fraction of 10^{-9} over the time period in question. To meet this goal from current achievements, two aspects need to be considered. The first is a careful test of the potential gyroscopic performance for a given large area, quantifying the relative frequency stability that is attainable within current technology. C-II, a well-engineered ring with $A = 1$ m^2, was built partly for this stability test, and the results[2,3] give vital insights for the mechanical design of G. C-II quantified the order of magnitude of the largest contribution to drifts of the Sagnac frequency δf in large rings. This arises from the dependence of backscatter-induced pulling on perimeter changes. Changes in the phase of backscattered light induced by perimeter variations strongly modulate the lock-in threshold under small changes in perimeter,[3,9] observed modulation periods in C-II including multiples of λ. The fact that this period appears to be tied to the length scale imposed by the optical wavelength means that pulling is changed by a larger fraction of the modulation amplitude when thermal and pressure-induced contributions to a perimeter change increase with P, modifying the above case. However, on the model used[1] the maximum lock-in threshold frequency l decreases as P^{-1} and the amplitude of the (modulated) relative frequency pulling is $1/2(l/\delta f)^2 \sim P^{-4}$, so that even these net pulling effects decrease rapidly with perimeter increase.

A complementary question is whether the further size upscaling from C-II to G—a 16-fold scaling of the area—is feasible for such gyroscopic applications or whether some new problem of principle appears, requiring changes from the basic principles of C-II design, particularly in view of the desired resolution. In this paper we illustrate the manner and extent to which such basic questions have been resolved by the successful design, construction, and operation of a test instrument: a square He–Ne ring laser called G0 with an area of 12.25 m^2. In continuation of the above summary, G0 proves that very large ring laser gyroscopes are feasible. Although quantitiative results are limited at this stage, all indications are that the performance of very large ring laser gyroscopes is indeed enhanced at the expected level. For example, one matter resolved by G0 is whether the technique used in C-II for restricting the ring laser to a single optical mode remains adequate at these higher dimensions. This method, called here the starvation technique, involves the servo-controlled reduction of the rf excitation power so that only one longitudinal mode (strictly, one pair of counterpropagating longitudinal modes)—that enjoying maximal gain—is excited. Single-mode operation is evidenced by the lack of a signal at the free spectral range (FSR) frequency; this correlates strongly with the Sagnac frequency having nearly its design value and also with the relative stability of the Sagnac frequency. Experience in C-II, for example, has shown that although multimode operation gives a rotation-related frequency, it is relatively unstable with respect to changes of rf power, making single-mode operation of critical importance for such gyroscopic applications.

The effectiveness of the starvation technique for mode selection can be expected to depend strongly on the size of the ring. The chosen technique of starving all modes but one becomes increasingly difficult with upscaling of the cavity. For some sufficiently high-perimeter P_c the technique must become inadequate, if only because the separation of adjacent longitudinal modes $f_{\mathrm{FSR}} = c/P$ decreases as the perimeter P increases, lowering without limit the gain difference for adjacent modes. For a Gaussian gain profile, for example, and with the surviving mode at its maximum,[1,10] the ratio of differential-to-absolute gain scales as P^{-2}. The effectiveness of the starvation technique was an imponderable in the design of C-I in the late 1980's and so was a substantial component of C-I's success. P_c was thus demonstrated to be greater than 4 m, but otherwise it is still quite unknown. Mode competition dynamics make a theoretical estimate of P_c a difficult task. An experimental test of the mode selection technique with $A \sim 16$ m^2 is vital. We present the outcome of such tests.

Other questions that need a full-sized ring for an adequate test include the following: How far is backscatter-induced pulling reduced in rings of these dimensions? (This depends to some extent on the angular distribution of the scattering at mirror defects. Although it is too early to give a definitive answer to this question, preliminary indications are encouraging for extrapolation to G; see Section 4.) What beam stability, beam profiles, waist sizes, and mode structure are expected of a G-sized cavity? (These are dictated by the geometry, principally the choice of mirror radii of curvature and also by the choice of gain tube geometry. The experience with G0 that is recorded here fully confirms standard expectations.) What is the optimal gain tube dimension in G? (Experience in C-II shows that when the internal diameter of the gain tube is 6 mm, transverse modes are excited because of the development of a local minimum in the gain profile on the axis. A gain tube with an inner diameter of 5 mm was chosen for G0 in the expectation of relatively large astigmatism in a cavity with marginal stability. In practice this proved to be compatible both with TEM(0, 0) excitation and with the absence of significant vignetting.) A fuller investigation is planned to consolidate these initial answers.

After the submission of this paper, the successful operation of a still larger ring laser gyroscope, a triangle with a perimeter of 40 m, was announced.[4] This success confirms the conclusions of the present paper and brings a new insight into the behavior of and potential for very large rings. Some relevant aspects are discussed at the end of Section 3.

In Section 4 we discuss some scaling arguments from our present experience for the potential of the instrument G and we provide a brief summary.

2. Design Constraints for G0

Where possible, the test ring G0 utilized components and strategies ported from the earlier projects C-I and C-II. This was partly to reduce both costs and time, but mainly to incorporate the benefits of the expertise acquired and the solutions adopted with these instruments.

A. Optical Characteristics

For the above reasons, the two curved mirrors used in G0 were taken at first from a batch designed for the much smaller ring C-II. This imposed two limitations. First, the mirrors that were used had losses greater than the mirrors installed in C-II (see Section 3). In addition, because of the perimeter change, the mirror radius of curvature (6 m) used for C-II is a factor ~3.5 smaller than is desirable for G0. Indeed, cavity stability is reduced in G0, and the beam profile has increased astigmatism, over the situation in which mirror radii are scaled with the perimeter. As in C-I and C-II these curved mirrors are diagonally opposed in G0, forming a square with two flat mirrors on the opposite diagonal (the diagonal configuration) in contrast with the cavity in which the curved mirrors are adjacent (the adjacent configuration). Although a diagonal cavity is asymmetrical, only then is G0 stable with these mirror radii (see Subsection 2.A.1). This indicates that the cavity stability is marginal with the only mirrors then available to us, and that better results may be expected with purpose-built mirrors with radii of curvature of the order of 21 m, which became available to us for the December runs. Henceforth the term G0 is taken to cover the diagonal choice of cavity geometry with mirrors of 6 m radius.

We show in Subsection 2.A.2 that, as might be expected from its marginal stability, use of nonideal mirror radius of curvature makes G0 especially sensitive to mirror misalignment. Hence G0 gives a particularly severe test of geometry constraints in a very large cavity, and its successful operation removes several obstacles in G design.

In Subsection 2.A.3 we give the waists, ellipticity, and position dependence of the beam profiles in G0. It is increasingly difficult to find a satisfactory compromise between beam vignetting and transverse-mode excitation as the perimeter increases. Again, G0's successful operation is encouraging for the prospect of very large rings.

1. Cavity Stability

A stability analysis of the strategy of using C-II mirrors in G0 is therefore a vital component of the design both to verify its feasibility and to specify the required tolerances and ranges for mirror alignment.

The stability of the diagonal configuration is most simply investigated by linearizing, which means unfolding a square cavity of side l at the flat mirrors to linear form so that the two curved mirrors are $2l$ apart, then using the standard linear stability criterion $|g_1 g_2| \leq 1$, where $g_i = 1 - 2l/R_i'$. Reference 11, which provides a more detailed analysis, gives the same criterion. For in-plane and out-of-plane stability, $R_i' = R/\sqrt{2}$ and $R_i' = \sqrt{2}R$, respectively. The in-plane criterion gives $R > \sqrt{2}l$. For G0, $R = 1.7l$, which is much closer to the threshold value of $1.41l$ than is C-I (where $R = 6l$). Hence a diagonal G0 cavity is marginally stable. An adjacent G0 cavity is unstable; using Ref. 11, the stability condition is $|\kappa| < 2$ where the in-plane stability $\kappa(\beta) = 3B^2 + 4B - 2$ and $B = 2(\sqrt{2}l/R - 1)$. This requires $R > 3\sqrt{2}l$, which does not hold.

2. Beam Steering

Increasing the perimeter of a ring laser introduces obvious demands on mechanical alignment. In addition, even with optimal mirror radii of curvature, a reduction in mirror alignment tolerances occurs because the optical lever arm length (over which an angular misadjustment affects the positions of beam spots) scales as P whereas the beam widths increase only as $\sqrt{P}$. In addition, use of nonoptimal mirror radii significantly magnifies in-plane beam steering effects. The design of G0 therefore had to include adequate allowance for meeting these revised tolerances. We estimated those from the theory published by Bilger and Stedman,[11] but with the correction that their definition of in-plane misalignment parameters should be replaced by

$$G_I = -c/s(\xi_{I+1} - \xi_{I-1}) + 2(l/s)(-\eta_I/R_I - \phi_I). \quad (1)$$

In Eq. (1) R_I is the radius of curvature of the Ith mirror, $c = \cos \pi/N$, $s = \sin \pi/N$ for an N mirror ring of side l. The Ith mirror pole is misadjusted by a displacement (ξ_I, η_I, ζ_I) with respect to local mirror axes (ξ toward the center of the ring and ζ out of plane) and also misaligned in the in-plane angle by a rotation ϕ_I (the out-of-plane angle does not contribute below). The in-plane beam walk parameters β are related to these misalignment parameters $\mathbf{G}$ by $D\beta = \mathbf{G}$. D is a tridiagonal matrix in which the diagonal elements $D_{II} = B_I = 2(l/sR_I - 1)$ and the nonzero off-diagonal elements are unity. The most severe tolerances for mirror alignment specified by this analysis relate to the in-plane angles and pole positions of the curved mirrors. For example, the angular misadjustment parameter ϕ_4 for a curved mirror in G0, when changed by misalignment by 1.0 arc min (0.3 mrad), gives in-plane a beam-spot motion on the same mirror of 4.8 mm. Given the target area as the beam width at the curved mirrors, i.e., ~2 mm, the maximal positional tolerances for the curved mirror poles are approximately 1 mm, and the maximal angular tolerances are approximately 10 arc sec (50 μrad).

3. *Beam Width and Shape*

Standard linearized Gaussian optics (pp. 597–617 of Siegman[12]) in a diagonal cavity gives a beam waist $w_0 = (\lambda L_R/\pi)^{1/2}$, where the Rayleigh length $L_R = l(R'/l - 1)^{1/2}$, R' is the mirror curvature radius corrected (as above) for astigmatism, and $l = P/4$. In G0 and C-II the out-of-plane waists are 0.98 and 0.74 mm, respectively, and the in-plane waists are 0.61 and 0.60 mm, respectively. Mode ellipticity is greatest in G0 where the axis ratio is 1.60 rather than 1.23. Although these waists in G0 are not greatly different from those in C-II, the beam width increases rapidly with distance from the waist so that the out-of-plane and in-plane widths are 1.44 and 0.90 mm, respectively, at the gain tube. This exacerbates not only the average beam loss but also the differential beam loss at the gain tube for counterpropagating modes and so the intrinsic asymmetry of a diagonal cavity.

This illustrates a design difficulty for all very large cavities including G0; the choices of gain tube diameter that induce unacceptable loss and those that stimulate transverse modes are squeezed together. For gain tube diameters $d \sim 4$–5 mm, vignetting is a significant risk, although waveguiding effects in the gain tube aperture will reduce these losses. (The effective mirror diameter is somewhat larger and not of concern here.) C-II experience shows that a gain tube inner diameter d of 6 mm stimulates transverse modes (the radial gain profile has a local minimum at the gain tube symmetry axis), that transverse-mode excitation can compromise Sagnac frequency stability, and that the mode purity and mode choice then become more sensitive functions of rf excitation power and of beam alignment in the gain tube and the beam combiner, etc. (The same reservations hold much more strongly for multimode operation.) The increase in beam ellipticity in G0 associated with its marginal stability is a disadvantage. For these reasons d was set to 5 mm for G0. However, beam losses from vignetting at the ends of the gain tube are then significant.

4. *Mode Structure*

We take the net Guoy phase as the sum of the in-plane and out-of-plane Guoy phases (as on pp. 643–647 of Siegman[12]). Hence the frequency shift δf_{mn} of a TEM(m, n) mode from the corresponding longitudinal mode, expressed as a fraction of the FSR $f_{\rm FSR} = c/P$, is $(2m + 1)\, h_i + (2n + 1)\, h_o$, where $h = -\tan^{-1}(P/4L_R)/2\pi$ and subscripts i and o denote in-plane and out-of-plane values of L_R.

In C-II, whose FSR is 75 MHz, $\delta f_{mn}/f_{\rm FSR} = 0.11m + 0.16n + 0.14$. In G0, $\delta f_{mn}/f_{\rm FSR} = 0.22m + 0.36n + 0.29$. Hence use of G0 increases mode frequency separations for a given FSR. The starvation method for mode selection relies on having relatively well-separated modes, one mode having a significantly higher amplification because of the increase in the gain profile at its frequency. Hence an increase in mode separations is helpful in obtaining single-longitudinal-mode excitation.

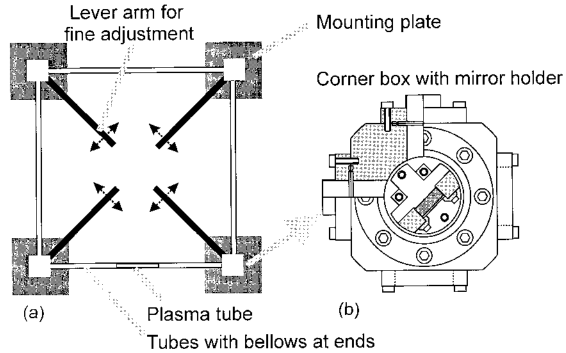

Fig. 1. Overall schematic of (a) G0 layout and (b) corner box detail.

B. Mechanical Characteristics

Again for reasons of cost and time, G0 is built of a relatively low-stability material (stainless steel). The chosen support has to be as stable as possible. A vertical concrete wall in the Cashmere cavern, which was built during World War II and is approximately 30 m underground, was selected, its height (3.5 m) dictating the perimeter and area of G0. Even in this location, substantial Sagnac frequency drifts from thermal variation of the concrete and steel are possible. It was regarded as a reasonable expectation, rather than essential for success of this test, that G0 should unlock and produce a Sagnac signal. However, larger rings of a given mirror quality are less susceptible to locking, and with reasonable mirror cleanliness G0 could be expected to unlock. The more optimistic expectations were justified.

The corners of G0 were set to a square matrix [Fig. 1(a)] with the sides vertical to within 1 mm and the diagonals at the design value to within 3 mm. In addition to meeting the demands of Section 2, this also aimed to define the G0 matrix with good repeatability and stability, the mirrors having their lowest loss at their poles. An all-metal system was used, but with the basic geometry of the ring defined by the concrete wall. Stainless steel was used for the tubes (40 mm in diameter with 20-mm diameter bellows) and boxes confining the laser gas. The corner boxes [Fig. 1(b)], 120 mm square and 90 mm deep, holding the mirror holders were themselves held by two H-section flexure units to 260-mm-square mild steel bases. The tilt of each box (and the mirror it contained) in plane and out of plane was adjusted using a lever arm of length 1 m whose end position was adjustable to a few micrometers using micrometer heads; this was done to meet the angular adjustability requirements of Subsection 2.A.2. The corner boxes and gain section were mechanically isolated from the pipes using stainless-steel bellows to minimize forces, to allow tilting, and to ensure that the concrete and not the metal parts dictated the mirror positions. Conflat connections CF 275 and CF 450 (and CF 133 for possible gas analysis) were used throughout, obviating *O* rings.

The gain tube (on the lower, horizontal leg) was approximately 100 mm long (with Kovar connections, etc.; the total length of this segment was 300 mm) and its internal diameter was set at 5 mm (see Subsection 2.A.3). Its transverse position was adjustable by four micrometer heads. An alignment technique similar to that used on C-I was applied. A beam from a green He–Ne laser was injected horizontally at the lower right curved mirror and the mirrors adjusted in a clockwise sequence by observing spot coincidence both near the mirror and 10 m away. When the spots from sequential round trips overlap within a given tolerance (i.e., 1 mm), their absolute position was also at the position of the alignment beam to this accuracy. Interference between the original and the circulated beams indicates adequate alignment for lasing to occur.

3. Results from Initial Operation

G0 was pumped down to a pressure of 10^{-8} Torr and leak tested and was then filled to a pressure of 3.5 mbars with helium and natural neon in a partial pressure ratio of 6:1. The rf discharge was created by a 3-turn coil wrapped around the outside of the gain tube, and a rf power of 5 W was adequate for lasing. The optical beams were detected with Hamamatsu red-sensitive photomultipliers. Initially (January 1998) this was performed on the clockwise beam rather than on a combined beam; this relied on the single-beam amplitude modulation at the Sagnac frequency induced by pulling.[9] The later results reported here and displayed in Figs. 2–4 (December 1998) were obtained with a diagonal cavity, with 21-m mirror radii, and through detection on the combined beam.

On completion, alignment, filling, and excitation, G0 lased immediately in TEM(0, 0). The FSR frequency of the cavity was measured by operating the laser in multiple longitudinal mode and detecting the resulting rf beats between modes with the photomultiplier. This frequency was found to be $F = 21.407$ MHz by heterodyning it with a local rf oscillator and corresponds to a perimeter P of 14.004 m, which is 3 parts in 10^4 different from the design value of 14 m and commensurate with the estimated precision for the installation.

Ring-down measurements yielded a cavity ring-down time up to 350 μs (depending on the alignment of the mirrors) and a cavity quality factor Q of up to 1.0×10^{12}. This implies an average mirror power loss $1 - R$ of 33 ppm. As expected from use of reject mirrors, this is below the level achieved in C-II and is not the ultimate for the instrument.

The predicted Sagnac frequency is

$$\delta f = \frac{4A\Omega}{\lambda P} = \frac{f_0 \Omega \cos \Lambda \cos \theta}{4F}, \qquad (2)$$

where the ring area A is $P^2/16$, the rate of Earth rotation $\Omega = 7.2921 \times 10^{-5}$ rad/s, for the 633-nm He–Ne line $f_0 = 4.736 \times 10^{14}$ Hz, and the latitude of the Cashmere cavern $\Lambda = -0.76056 = 43.5679° = 43°34'37''$ S, the cavern wall being mounted at $\theta = 0.174 = 10.0° \pm 0.5°$ to the East–West line. This yields

$$\delta f = 287.75 \pm 0.46 \text{ Hz}. \qquad (3)$$

The error here reflects uncertainty in the cavern wall orientation. The comparison with experiment is also limited by frequency pulling (see below).

The dc component of the photomultiplier signal was used as a measure of the optical output power. Single-longitudinal-mode operation was maintained by stabilizing this dc signal, adjusting the rf drive power using a proportional-integral servo controller with a time constant of 1 s. The ac signal was passed through an antialiasing filter and digitized at 1000 samples/s. Postprocessing filtering in the 283–291-Hz band removed harmonics and mains (50-Hz) interference.

When the servoed power level was reduced, the FSR signal disappeared cleanly into the −115-dBm noise floor of the rf spectrum analyzer, which gave a quantitative demonstration of single-mode lasing. This tested the heart of the starvation technique described in Section 1; because the differential gain required for single mode decreases quadratically as the perimeter increases, and the emergent beam powers in C-II are as low as 10 pW, the demands are high. The anticipated problem was not the fineness of control; the laser beam power is a steep function of rf power, and it is relatively easy to servo the latter from the former. The problems anticipated included, for example, that the threshold rf powers for single-mode lasing and multimode lasing might coalesce, making the single-mode regime unavailable in very large rings. In this case, G0 yielded a clear single-mode regime of operation. A Sagnac signal was then found. Initially (January 1998) this had a frequency of 287.3 Hz, drifting by ~1 Hz in 1 ks, and in later data this was 288.8 Hz and with a tenth of the earlier drift rate. The frequency estimates were made using a second-order autoregressive model. The signal was 45 dB above the noise floor when the whole cavern was made as quiet (mechanically and electrically) as possible. The Allan standard deviation of the Sagnac frequency estimates in January 1998 exhibited a minimum at 7×10^{-3} Hz for a sampling time of approximately 3 s, and in December 1998 a similar knee shifted to higher times. Figures 2, 3, and 4 document some data from 2 December 1998 as an illustration. The Sagnac spectrum is taken from the modulation of a single beam, and the depth of this modulation is 1–2%; this itself is evi-

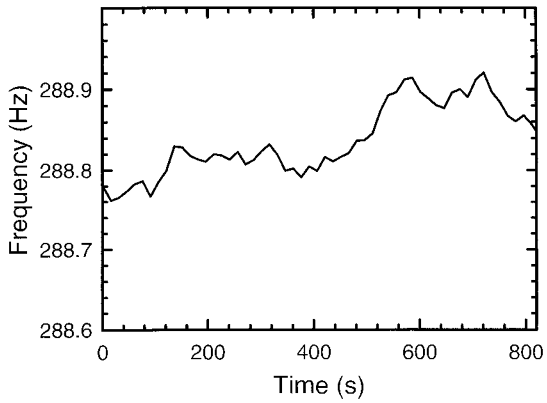

Fig. 2. Drift in the Sagnac frequency with time for G0 runs GNE35-36, 2 December 1998. The frequency estimates were made using a second-order autoregressive model.

dence for frequency pulling effects as a consequence of backscatter. In the standard model (see, for example, Stedman *et al.*[9]) the modulation depth for single-beam intensity in the dissipative regime, like the ratio of adjacent harmonic amplitudes, is of the order of the ratio of the lock-in threshold frequency to

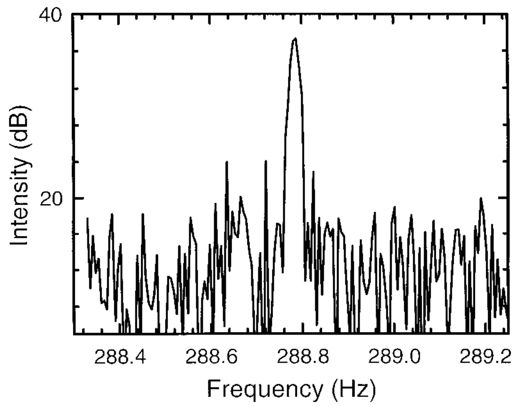

Fig. 3. Raw Sagnac frequency spectrum for part (327 s) of the run of Fig. 2.

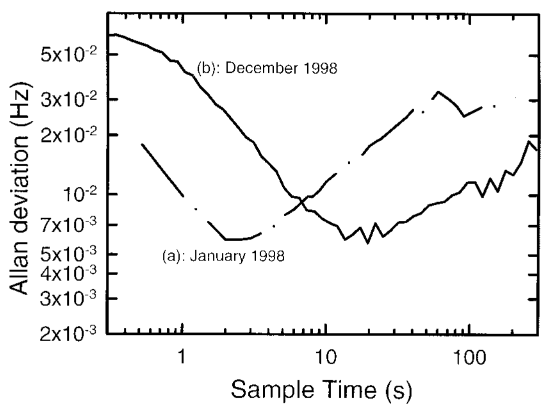

Fig. 4. Allan deviations in G0 as a function of sample time. These values were derived from the autoregressive Sagnac frequency estimates. (a) The run of 29 January 1998 and (b) the run of Fig. 2 on 2 December 1998.

the Sagnac frequency. This suggests a lock-in threshold frequency $l \sim 1$–2 Hz. The Aronowitz criterion,

$$l \sim \frac{c\sqrt{1-R}\lambda}{8\pi dP}, \tag{4}$$

where d is a beam diameter (typically 1 mm) and R is estimated as above from the ring-down time, yields $l \sim 25$ Hz. This in turn yields a frequency pulling of $\sim$1 Hz, consistent with our observations in G0.

As stated in Section 1, we now comment in the light of experience with G0 on some aspects of results recently announced[4] from a triangular ring gyroscope with a perimeter of 40 m. This triangular ring has mirrors with a power reflectance nominally of 99.7% instead of the 99.997% value consistent with the measured ring-down time of G0, and it also incorporates Brewster windows. However, its size also permits it to unlock under Earth rotation with a Sagnac frequency of approximately 492 Hz. Its single-mode operation with output power of the order of 100 μW is facilitated by the strong inhibition of lasing of adjacent longitudinal modes, despite the FSR being of the order of 7.5 MHz. The suppression region has a width of 200 MHz, which Dunn[4] attributes to the width of the pressure-broadened line. Our experience is that, for a variety of gas pressures, longitudinal modes that are adjacent at the G0 FSR of 21 MHz are readily excited at higher powers, although there is certainly a complicated pattern of behavior of FSR under power cycling, with near-discrete changes in the Sagnac frequency at the threshold powers.

4. Discussion

Single-longitudinal-mode operation and an unlocked Sagnac frequency from Earth rotation have been demonstrated in a vertical ring laser gyroscope with a perimeter of 14 m. Further studies of this system will be of considerable interest in their own right and also for the detection of vertically polarized seismic waves. Above all, these achievements give encouragement, complementing the frequency stability results from C-II, that a geodetic ring laser G of similar area is practical.

We now combine our experience with a well-engineered large ring C-II[3] and a very large, if simply, constructed ring G0 (discussed in this paper), noting also the report of a ring of perimeter 40 m,[4] to make a quantitative estimate of the value of upscaling (whose advantages in principle have several origins[1]) and in particular of the potential sensitivity of a well-engineered very large ring laser such as the system G now in the design phase.

First we define the goal. G will be commissioned for geodetic purposes to measure short-term fluctuations (with periods of hours to days) in the rate Ω_E of Earth rotation.[1] Short-term fluctuations in Earth rotation, at the level of parts in 10^9, are relatively poorly known.[5,7,8] A primary requirement of any sensor of Earth rotation fluctuation is that instrumental drift in the measurement of the rotation rate should be less than a fraction 10^{-9} over the time period in question, and the relative error in the scale factor ($G = 2\pi\delta f/\Omega$ by which an angular rotation rate Ω is amplified to yield an angular Sagnac frequency $2\pi\delta f = 8\pi A\Omega/\lambda P$) is limited to parts in 10^9 per day.

From experience with C-II and G0, the major cause of scale factor variation is expected to be the variation in Sagnac frequency δf with perimeter P, arising from backscatter-induced changes to frequency pulling. The variation is roughly sinusoidal, defined by an amplitude of frequency excursion $\delta f'$ over the pulling cycle and by the pulling period Π (that is, the perimeter variation inducing one cycle of change in δf). Π is linked to the wavelength; we write $\Pi = M\lambda$. Hence taking the Sagnac frequency variation to be of the form $\delta f' \sin(2\pi P/M\lambda)$, the coefficient for the variation of Sagnac frequency with perimeter variation as a fraction of the Sagnac frequency is

$$\Xi = \frac{1}{\delta f}\frac{d\delta f}{dP} \sim \frac{2\pi}{M\lambda}\Upsilon, \Upsilon = \frac{\delta f'}{\delta f}. \quad (5)$$

In a previous report on C-II, $\delta f' \sim 2$ Hz and $M \sim 3$, so that from Eq. (5), $\Upsilon \sim 0.025$ and a mean value for $\Xi \sim 8 \times 10^4\ \mathrm{m}^{-1}$; for more favorable parts of the pulling cycle, $\Xi \sim 2 \times 10^4\ \mathrm{m}^{-1}$.[3]

To extrapolate from these measurements to a soundly based prediction for the frequency stability of G requires estimation of the frequency pulling amplitude $\delta f'$, of the pulling period parameter M, and of the likely perimeter change δP. The magnitude of $\delta f'$ can be assessed on the Aronowitz model, in which $\delta f' = \delta f - p$, where p is the pulled frequency $(\delta f^2 - l^2)^{1/2}$. Hence from approximation (4) the relative pulling coefficient is

$$\Upsilon = \delta f'/\delta f \sim 2(1 - R)(c\lambda^2/32\pi d A\Omega_E)^2. \quad (6)$$

Hence for a given mirror finesse, Υ scales as P^{-4} and (for a given M as well) Ξ scales as P^{-4}. The measured Q of C-II corresponds to an average mirror loss $1 - R$ of 17 ppm, which from Eq. (6) corresponds to $\Upsilon \sim 1\%$, which is of the same order as the observed value for Υ given above. As discussed above, in G0 the measured ring-down time and Sagnac frequency drift both suggest a frequency pulling magnitude of $\sim$1 Hz. We use in our estimates a mirror loss that is the geometric mean of the design loss (1 ppm) and that observed in C-II thus far (17 ppm), or 4 ppm. With these choices, Eq. (6) yields $\Upsilon \sim 7 \times 10^{-4}/A^2$, or 700 ppm in C-II. Importantly, the P^{-4} scaling of the relative pulling coefficient in Eq. (6) yields $\Upsilon \sim 4 \times 10^{-5}$ in G.

Schreiber *et al.*[3] gave reasons for expecting a significant increase in the pulling period parameter M through use of more symmetrical mirror holders in C-II. Values of $M \sim 10$ have been documented in other rings. In unpublished data from C-II during December 1998, after a change of cavity geometry to a more symmetrical form as suggested by the analysis of Schreiber *et al.*[3] (but no change in Q), the sensitivity of the Sagnac frequency to (now uniform) dimensional changes has been reduced by a factor of 25–100, the ratio of Sagnac frequency change to FSR change in one run being 0.017; we therefore take $M \sim 50$ below.

In C-II the major source of perimeter instability is that of ambient pressure variation. In G the pressure will be stabilized to 0.1 hPa (1 part in 10^4). If we assume that the mirror mounts increase the effective bulk compressibility by that of bulk Zerodur, this implies a maximum perimeter change δP in G of the order of 1 nm or $1.5 \times 10^{-3}\ \lambda$. Hence a reasonable prediction for the relative frequency stability of G is $\Xi\ \delta P \sim 1 \times 10^{-8}$. More directly, in this December 1998 run from C-II that also monitored FSR, the ratio of Sagnac frequency change to pressure change was 4 μHz/Pa and $\Xi \approx 430\ \mathrm{m}^{-1}$. This yields for G a value $\Xi \sim 1.7\ \mathrm{m}^{-1}$, and so a relative frequency stability in G of $\Xi\delta P \sim 2 \times 10^{-9}$.

This estimate does not take into account or compensate for such drifts as instrumental effects through modeling. This argument, which is based throughout on observed parameters in rings of varying size, notably G0, supports the expectation that it is feasible for G to enter the regime of measurement of Earth rotation fluctuations.

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