Development of an Optical Gyroscope for Ground Tilt Measurements in Advanced LIGO

Progress Report 1

Michelle Stephens
Mentors: Alastair Heptonstall and Peter King
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Laser Interferometer Gravitational Wave Observatory
- LIGO -
California Institute of Technology
Massachusetts Institute of Technology

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1. Introduction

Gravitational waves were first predicted as a consequence of Einstein's General Theory of Relativity. These “ripples in space-time” are quadrupole emissions given off in response to the change in position of some massive astronomical bodies, such as binary neutron stars, and propagate through space at the speed of light. The Laser Interferometry Gravitational-wave Observatory (LIGO) is a collaboration of more than 600 scientists whose goal is to detect these gravitational waves.

Three main detectors comprise LIGO. Two of these are collocated in Hanford, Washington, one with 4 km arms and a second inside of it with 2 km arms, although the 2 km detector is not currently being used. The third 4 km detector is located in Livingston, Louisiana. Having detectors in different locations is necessary in order to confirm the event of a gravitational wave detection, and to be able to locate the region of the sky from which the source originated [8]. The detectors operate as Michelson interferometers, and a simple conceptual model for the way they work is this: if a gravitational wave passes by, one arm will be stretched while the other will be contracted, and this relative motion between the test-mass mirrors at the end of each arm will cause a change in the interference pattern at the output.

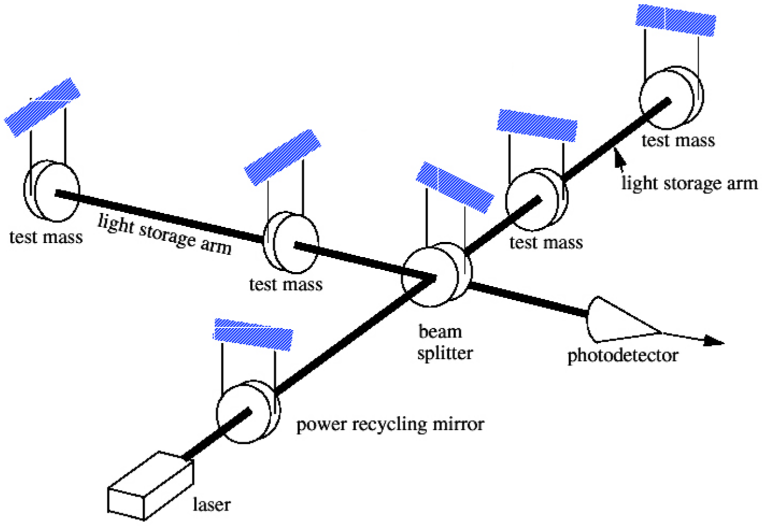

Fig. 1. Diagram including the main components of the LIGO interferometers [9].

Initial LIGO first took data in 2002, and has had five science runs since that time. In November of 2005, at the beginning of the fifth science run, Initial LIGO reached its design sensitivities, however, noise still dominated the signal, and to-date no gravitational waves have been detected [9]. Advanced LIGO is a planned upgrade to the detectors which will increase sensitivity by a factor of about 10, and will listen to a volume of space 1,000 times greater than that of Initial LIGO [10]. Much of the ongoing work in LIGO is focused on decreasing the various noise sources in the detectors, such as seismic noise arising from the motion of the ground, thermal noise in the mirrors, laser intensity (radiation pressure) noise, and shot noise. Together, the radiation pressure and the shot noise comprise quantum noise. Decreasing the shot noise involves increasing the laser intensity, which increases the radiation pressure. In this way, there is a fundamental limit to the quantum noise called the Standard Quantum Limit.[1] Advanced LIGO ultimately aims to be limited by quantum noise.

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[1] It is possible for the detectors to operate below the Standard Quantum Limit by using ‘squeezed’ states of light. For a further discussion of this, see A. Heptonstall’s “Characterization of Mechanical…” p.22 [6].

The consequences of being able to detect gravitational waves are tantalizing. Scientists would be capable of seeing a huge volume of the universe that is currently dark to us, and could observe the most energetic events happening in it. The processes that continue to form the structure of our universe – mergers of black holes, supernovae – could be seen in an entirely different spectrum. We may see things that are unfathomable and unpredictable from our current perspective. Much like the advent of radio or infrared astronomy, gravitational wave observatories will be a whole new window into understanding the cosmos.

2. Project Objectives

2.1 Background

One of the major noise sources in the LIGO detectors is due to the motion of the earth's crust – from things like seismic activity, tidal deformation, and simply cars driving on nearby freeways. Advanced LIGO's test masses will be suspended as pendulums from 400 µm quartz fibers, which are as strong as steel. This takes advantage of an interesting property of pendulums: at frequencies above their resonant frequency the pendulums are very effective at filtering out ground motion. Above resonance, the motion of the test masses falls off as $1/f^2$ [6]. It is at and below the resonant frequency of the pendulum that displacement of the test masses becomes problematic.

Seismometers at the detector sites are capable of measuring the motion of the ground very accurately, and they feed this signal forward to hydraulic HEPI actuators, which then make corrections for this motion. This system is good enough to achieve design sensitivity at frequencies above 10 Hz. At very low frequencies, however, ground tilt is coupled into the horizontal seismometer signal, which may cause the HEPI actuators to make erroneous corrections. Thus, some way of subtracting out this rotation component is necessary to reduce the noise in the detectors at low frequencies. This will also help with detector stability, reducing the gain needed to keep them locked. This in turn will make the detector more sensitive at higher frequencies. The focus of this project is to build a laser-based optical gyroscope capable of sensing ground tilt very accurately.

2.2 Current Technology

Laser-based gyroscopes operate on the Sagnac principle, where the path lengths for counter-propagating beams traveling around a rotating ring are different, yielding a beat frequency at the output of the cavity that is proportional to the rotation rate.

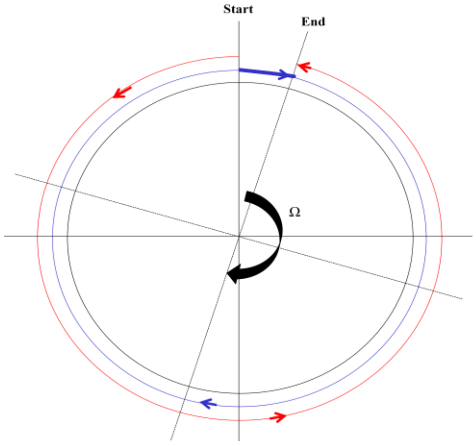

Fig. 2. Illustration of the Sagnac Effect

Because the output signal of a Sagnac interferometer is proportional to the relative speed of the test masses rather than their position, Sagnac interferometers are capable of operating below the Standard Quantum Limit [6].

Laser gyroscopes are commonly used in inertial guidance systems for military applications, and in some consumer items. These, however, are generally required to measure large, fast rotations, and do not require the high sensitivity that we need.

3. Building a Passive Laser Gyroscope

3.1 Requirements

For a horizontal seismometer, the ratio of sensitivity to rotation to sensitivity to horizontal motions at frequency ω is given by [1]:

$$\frac{rotation\ sensitivity}{horizontal\ sensitivity} = -\frac{g}{\omega^2}$$

To find the required sensitivity to rotations, it will be assumed that noise from the rotation sensor can contribute only 1/10th of the total noise in the horizontal direction. This will ensure that the seismometer's signal is not dominated by tilt, and the HEPI actuators will not make erroneous corrections. This can be expressed as [1]:

$$\Omega_{sensitivity} = \frac{1}{10}\frac{\omega^2}{g}x_{sensitivity}$$

Where $x_{sensitivity}$ is the horizontal sensitivity requirement. At 0.2 Hz, this gives a rotational sensitivity requirement of 3×10^{-9} rad/√Hz.

3.2 Design

We are not as limited by the size constraints that commonly accompany units used in other applications. Our research is focused on a passive setup (the lasing medium is outside the cavity) using fixed mirrors.

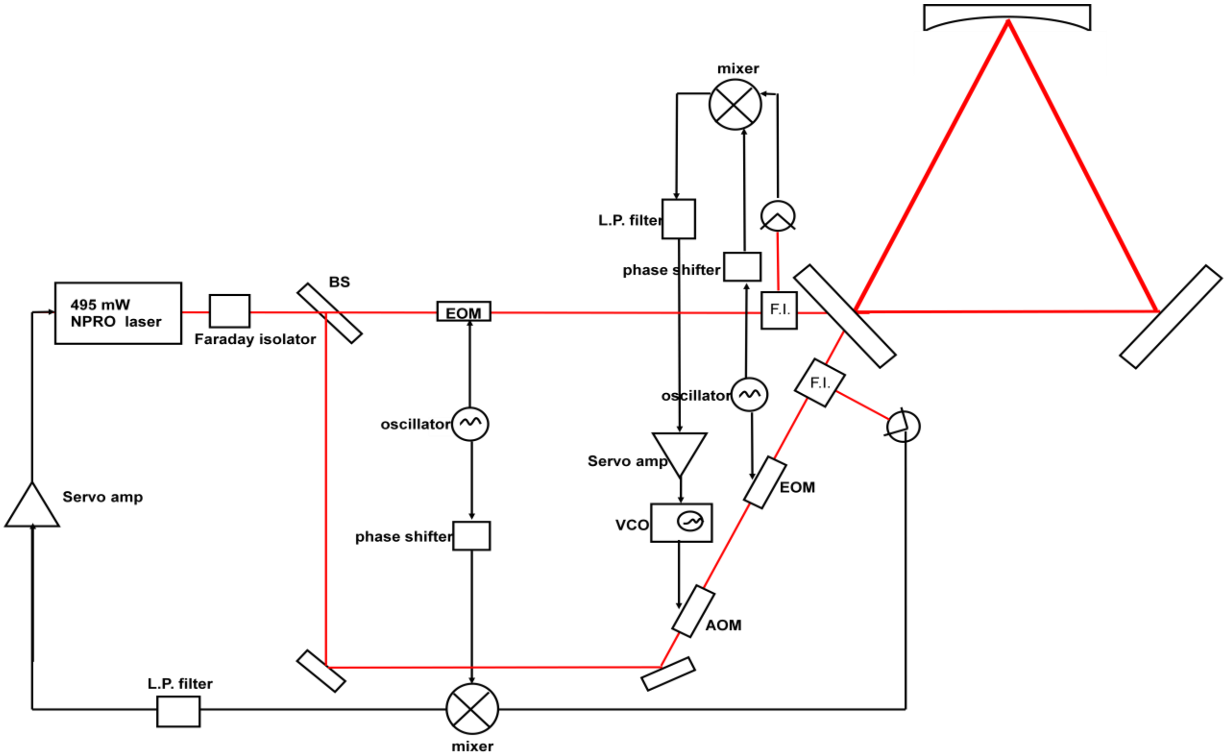

Fig. 3. Diagram of an externally excited passive laser gyroscope.

1064 nm light from a 495 mW NPRO laser will be split into two counter propagating beams, which will each be locked to the cavity using the Pound-Drever-Hall locking method.[2] Both beams will be modulated using an Electro-Optic Modulator (EOM) to produce sidebands, which allow the lasers to be locked. The sidebands are reflected from the cavity, acting as stable references that sample the phase of the main carrier. Thus, the EOM modulation frequency must not be resonant with the cavity. Each side of the triangular cavity will be 1 m in length, giving a free spectral range of 50 MHz. We will modulate the EOMs at 10 MHz.

The counterclockwise beam will be locked to the cavity by acting directly on the laser to alter its frequency. The clockwise beam will then be locked to the counterclockwise mode, using an Acousto-Optic Modulator (AOM) to alter its frequency.

Our readout point will be the modulation frequency of the AOM, which is proportional to the rotation rate of the ring.

3.3 Finesse

The finesse of the cavity is completely determined by the losses in the mirrors, where 1- ρ is lost, according to:

$$\mathcal{F} = \frac{\pi}{2\sin^{-1}\left(\frac{1-\sqrt{\rho}}{2\sqrt[4]{\rho}}\right)}$$

A higher finesse means lower energy loss in the cavity. It is also inversely proportional to the full width at half-maximum bandwidth of the cavity resonances, with a proportionality constant equal to the free spectral range of the cavity. Thus, a higher finesse denotes a smaller range of frequencies at which the laser will resonate in the cavity, and in our system, results in a more stable laser.

The triangular cavity consists of two mirrors with a reflectance of 99.99% and one partially transmitting mirror with a reflectance of 99 ± 0.5 %. This yields a finesse of approximately 1000.

3.4 Sensitivity Limitations

While the error signal is a measurement of laser frequency noise, it is not possible to distinguish between this, and length noise in the cavity. Length noise in the cavity can be reduced by using stable optical mounts for the mirrors [4]. The whole system is being locked on resonance, so the carrier is not reflected. Because of this, variation in the laser power, the modulation amplitude, and the modulation frequency do not contribute to the error signal [2]. Fluctuations in the sideband power will contribute to the error signal, however, this effect falls off as the frequency increases [3].

We expect a few other sources to contribute to noise in the error signal, including the vibrations of mirrors and thermal expansion of the optical table. Ultimately, however, we expect that our laser gyroscope will be limited in sensitivity by shot noise. The sensitivity of a shot noise-limited passive laser gyroscope is given by [5]:

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[2] See appendix for discussion of the Pound-Drever-Hall Technique

$$\delta\Omega = \frac{\lambda P}{4A}\frac{\sqrt{2}\Gamma}{\sqrt{(n_{ph}\eta\tau)}}$$

Where P is the perimeter of the ring, A is the area enclosed therein, Γ is the full width at half maximum bandwidth, n_{ph} is the number of photons arriving at the detector, η is the detector efficiency, and τ is the integration time. For a cavity with 1m sides, a finesse of 1000, and the maximum power output of our laser, the sensitivity will be 2.5 x 10^{-8} rad/√Hz.

4. Experimental Progress

This project is currently in its beginning stages. The first step we will take, before proceeding with the design of the laser gyro, will be focused on building and locking a simple Fabry-Perot cavity using Pound-Drever-Hall techniques. When we add the third mirror for the triangular cavity, we will already have a working control system, and a working modulation and demodulation system.

4.1 Characterization of an NPRO Laser

The power output as a function of drive current for a 1 W NPRO laser was measured. This laser is housed in the same lab as our experiment, and although this is not the laser we will be using for our gyroscope, the measurement will need to be repeated for the 495 mW NPRO laser as soon as the Standard Operating Procedure (SOP) is approved. The results were plotted in Matlab.

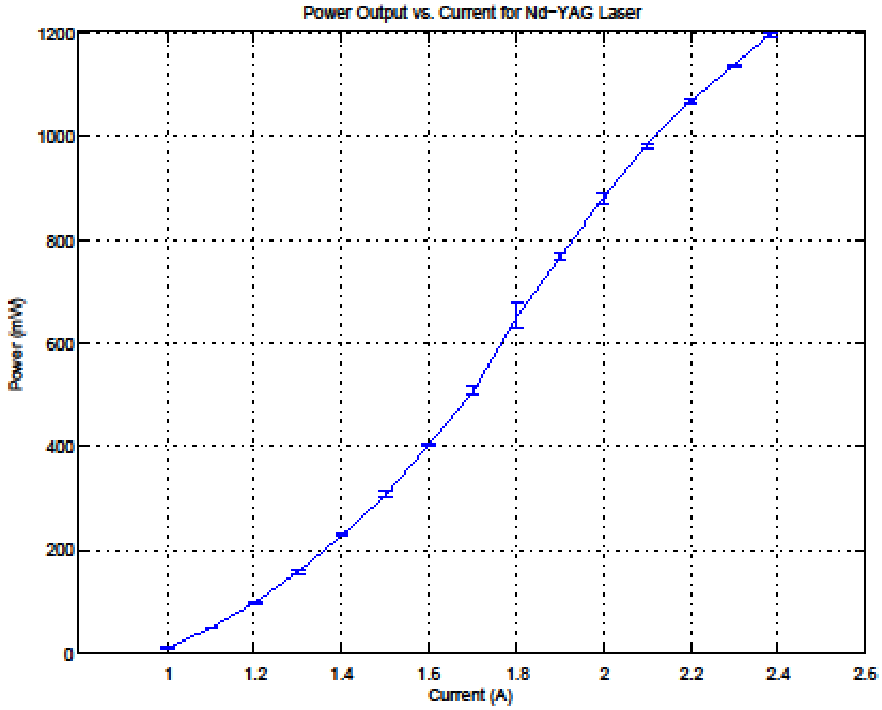

Fig. 4. Power output for a 1 W Nd-YAG laser.

Error bars denote the range over which the power fluctuated at each value for current.

4.2 Divergence Measurements of an NPRO Laser

The 1W NPRO laser operates in the fundamental transverse mode and has a Gaussian profile. Gaussian beams diverge with distance according to [7]:

$$w(z) = w_0 \sqrt{\left(1 + \left(\frac{\lambda z}{\pi {w_0}^2}\right)^2\right)}$$

where w_0 is the half-width of the beam at its narrowest point, called the *waist*, and *w(z)* is the half-width of the beam at a distance *z* from the waist, as shown in the figure below.

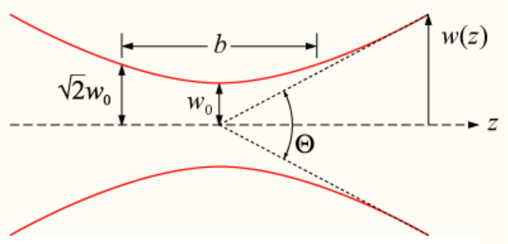

Fig. 5. Contour of a Gaussian beam.

The asymptotes in figure 4 correspond to [7]:

$$\Theta = \frac{\lambda}{\pi w_0}$$

where a small angle approximation has been made. It is necessary to know how the beam of our laser diverges, so that we can avoid diffraction losses in the cavity and the other optics of our setup by correctly refocusing the beam. The measurements made for this laser will be repeated on the 495 mW laser.

Measurements were taken using a BeamScan XYS-10A, which profiles the beam along two perpendicular axes. The full width at which the intensity drops to $1/e^2(I_{max})$ was found as a function of distance from the laser aperture. The data was plotted using Matlab and it was found that the beam's divergence was linear, suggesting that it had reached its asymptotic value. It is likely that the output coupler of the laser just before the aperture causes this rapid divergence. The size of the beam's waist was found by measuring the slope of the data, and approximating it as Θ. The characteristic divergence iwas then fit to the data by using a least-squares linear regression in Matlab. The results of this are shown in figure 6.

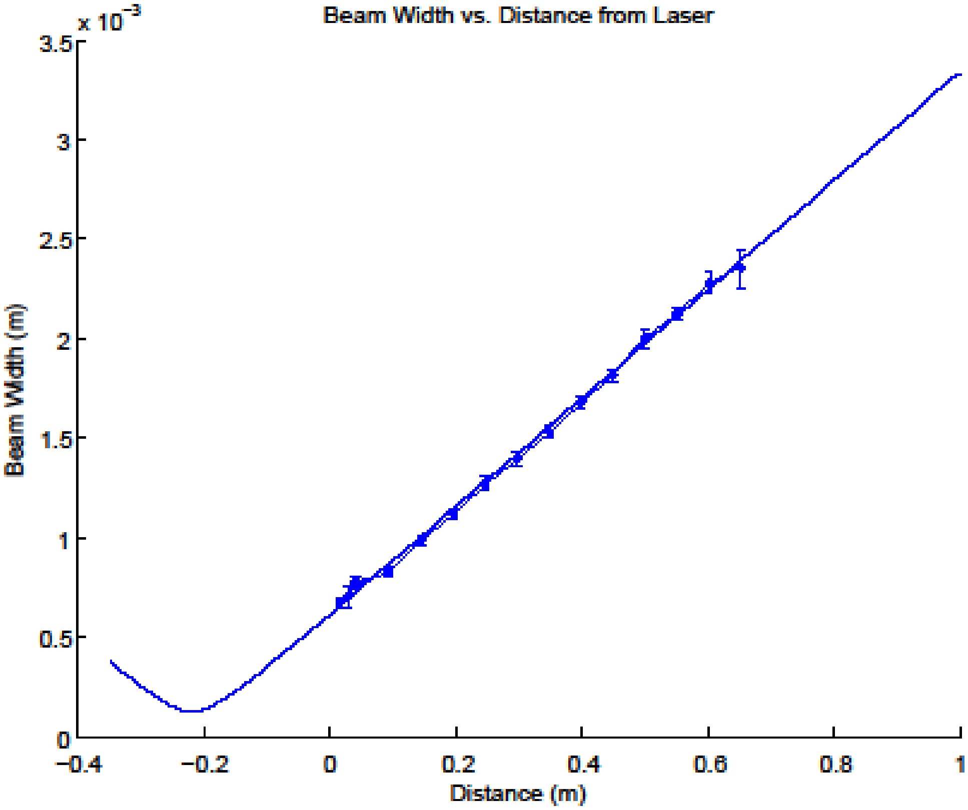

Fig. 6. Beam divergence of a 1 W Nd-Yag laser. The waist is located 21.5 cm behind the laser aperture, with a half width of 124 micrometers.

4.3 Simple Cavity Optical Setup

Before beginning to set up and lock the triangular cavity, we will try to lock a simple cavity using Pound-Drever-Hall.[3] We blocked out the components of this setup on the optical bench and will begin alignment as soon as the SOP is approved. A picture of this setup is shown in figure 7, with the integral optical components labeled.

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[3] The setup for this is shown schematically in Appendix A.

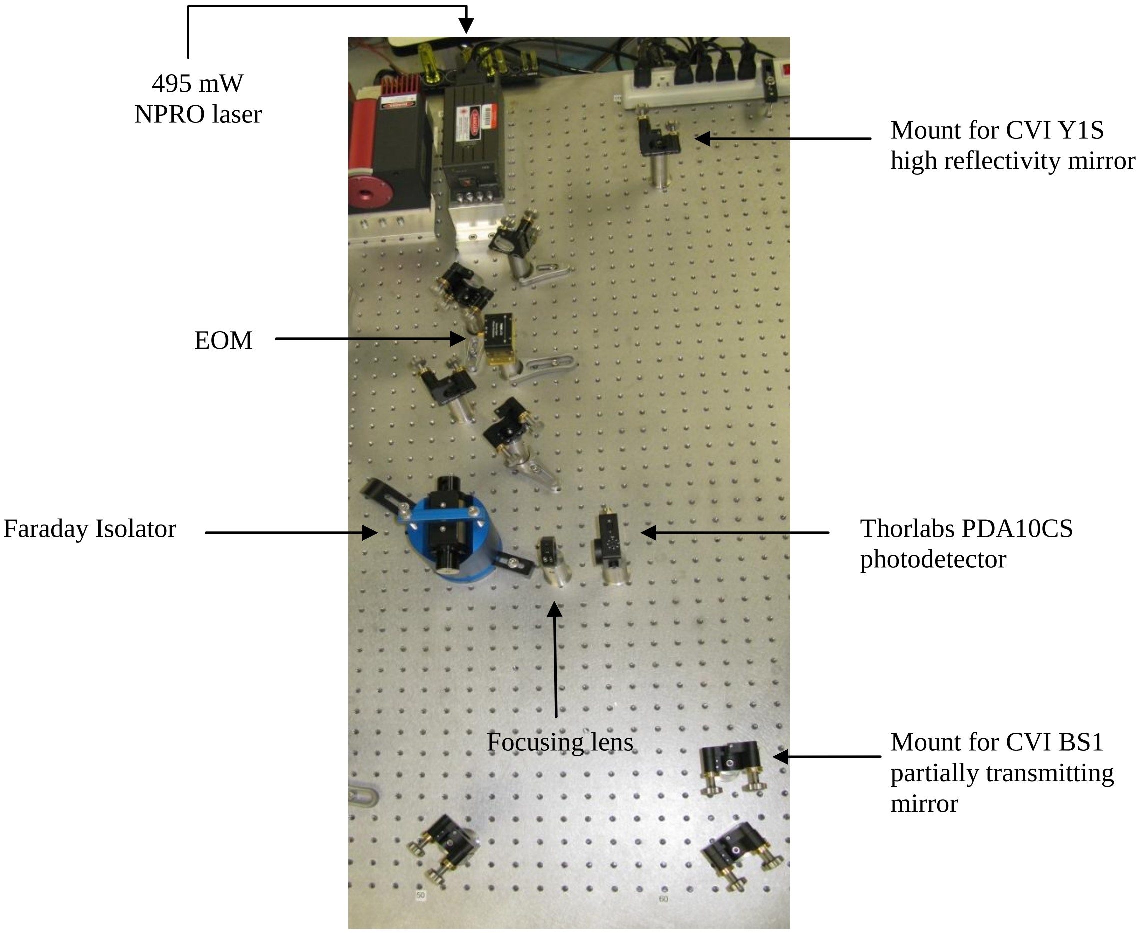

Fig. 7. Initial mock-up of PDH setup.

We want to use the same modulation frequency for the EOM in this setup, in order to align as closely as possible what the conditions will be for the actual setup. The length of the Fabry-Perot cavity is 1 m, and its free spectral range is 150 MHz. If we drive the EOM at 10 MHz, the sidebands will not be on resonance with the cavity.

5. Schedule

Below is the original timetable for the project.

1.5 Weeks	Theory and calculations
2 Weeks	Ordering/making parts
2 Weeks	Locking a simple cavity
2 Weeks	Setting up/locking the triangular cavity
2.5 Weeks	Troubleshooting

We are currently waiting for the standard operating procedure for our laser to be approved, and for the delivery of beam steering optics needed for our setup. We should move into locking a simple cavity by the fourth week, thus we are on schedule with this timetable.

6. References

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Appendix A. Pound-Drever-Hall Locking

Pound-Drever-Hall locking [2, 3] is a technique used in several parts of the LIGO detectors because it is a very fast feedback mechanism and produces a very frequency-stable laser. “Locking” in this context refers to either altering a laser's frequency to be an integer multiple of a cavity's free spectral range, or altering the length of the cavity to match the laser's frequency. We will be utilizing the former of these two techniques, and a simple schematic diagram of this is shown below.

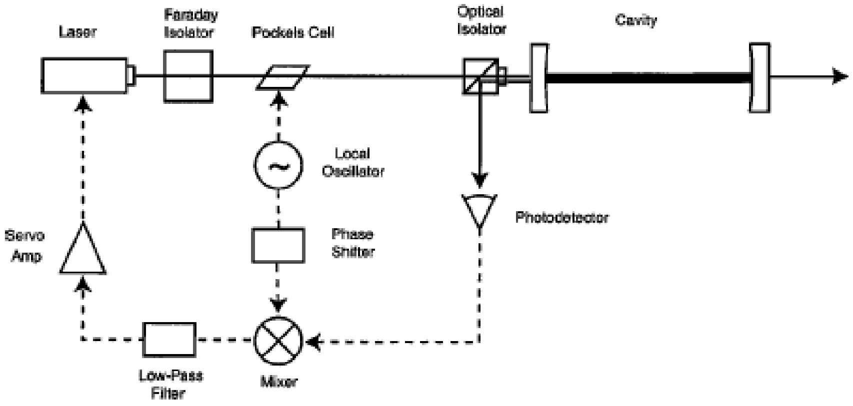

Fig. 8. Setup for locking a laser to a cavity [3]

This system is set up in such a way that it will hold the reflected intensity at zero; hence, it will keep the laser’s frequency on resonance with the cavity. Although the reflected intensity is symmetric about resonance, this system samples the derivative of the reflected intensity. This is done by dithering the frequency and gauging the response of the reflected beam. In the setup above, the frequency is modulated with a Pockels Cell, which is labeled as an EOM in our setup. The Pockels Cell is modulated at a known frequency with the local oscillator. The reflected beam is sent to the photodetector via an optical isolator. The signal from the local oscillator and the photodetector are mixed, and the output contains a low frequency (dc) signal, and a signal at twice the modulation frequency. The dc signal is what samples the derivative of the reflected intensity, thus a low-pass filter is placed between the mixer and the servo amp which will feedback to the laser. The phase shifter in the diagram is used in practice to compensate for unequal delays in the signal paths, since we need to mix two sinusoids which are in phase with each other.

Below is the characteristic error signal of Pound-Drever-Hall locking. The error signal is linear near resonance, allowing any change in the frequency to be compensated for.

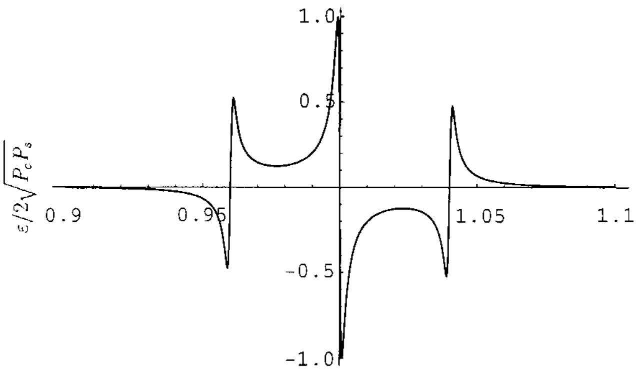

Fig. 9. The Pound-Drever-Hall error signal [3]