

# Modelling of seismic noise at 40-meter interferometer

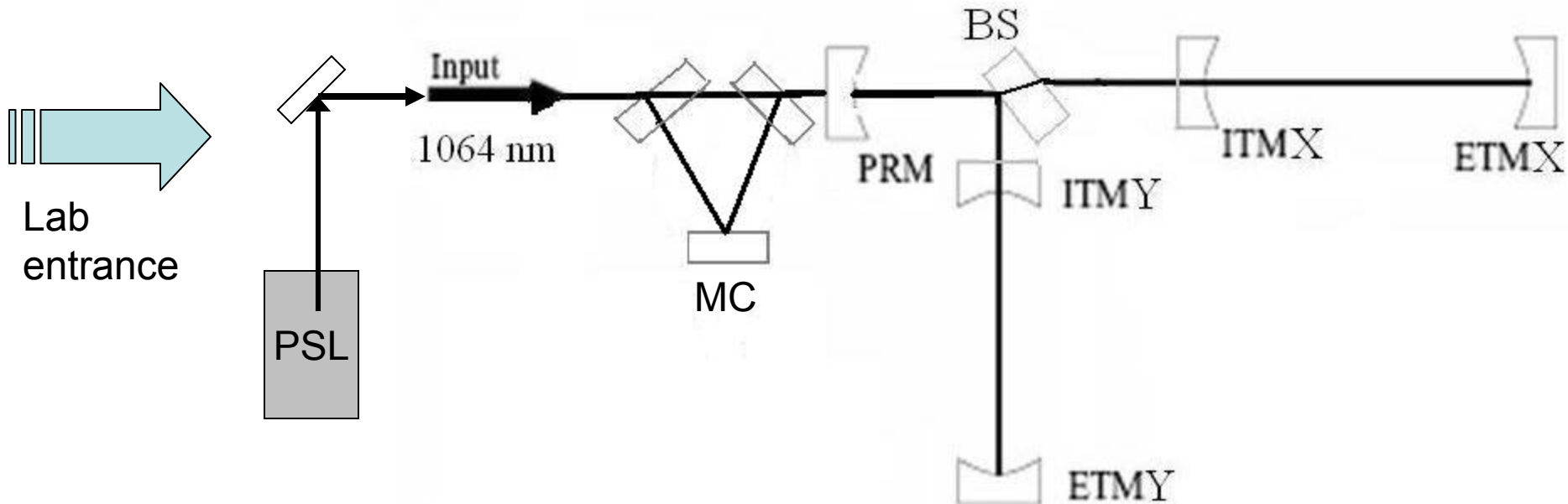


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November 2007

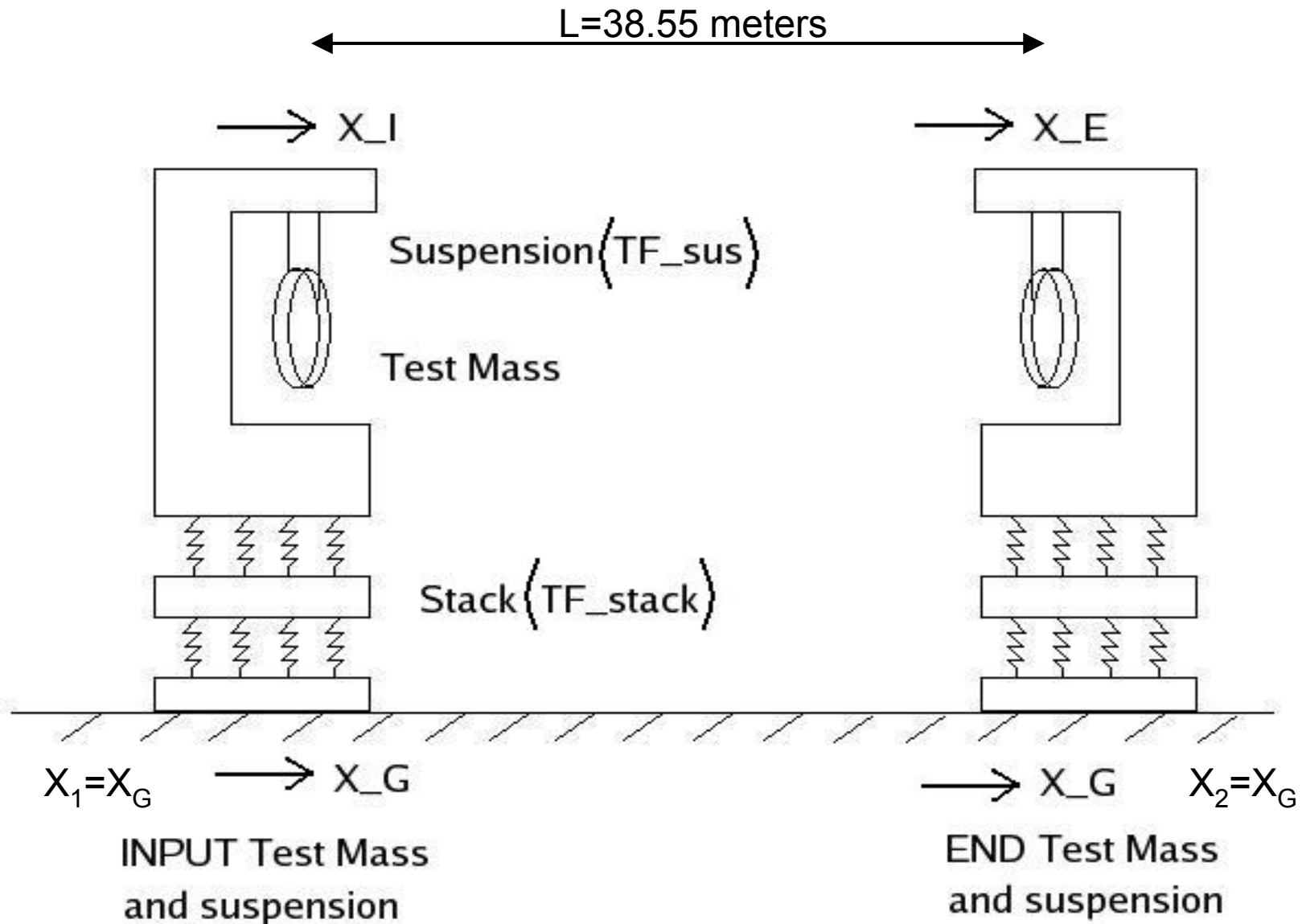
# OUTLINE

- ➔ General explanation: what is LIGO 40-m
- ➔ Discussion of transfer functions
- ➔ Calculation of the differential length of the X-arm as a function of Q-factors of “input” and “end” test masses suspensions
- ➔ Short discussion of the result
  
- ➔ “Bonus-tracks”: Applications of LIGO 40-m tripod not in physics

# What is 40-m interferometer ?



# What am I modelling ?



$X_1 = X_2 = X_G$  (noise motion of the absolutely rigid concrete slab)

# Transfer functions

General definition (what mathematical expression to treat as transfer function?)

Equation of motion for harmonic oscillator:  $m \ddot{x}(t) + b \dot{x}(t) + kx(t) = F(t)$

two approaches:

(1) force  $\rightarrow$  displacement,

Laplace transform

$$\frac{x(s)}{F(s)} = \frac{1/m}{s^2 + \frac{b}{m}s + \omega_n^2};$$

$$s \leftrightarrow j\omega$$

$$x(s) = F(s) \left( \frac{x(s)}{F(s)} \right)$$

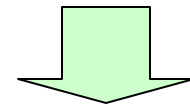
$$x(s) \rightarrow x(t)$$

(2) displacement  $\rightarrow$  displacement

$X_i$  is input to the system,

$X_0$  is output, position of the test mass,

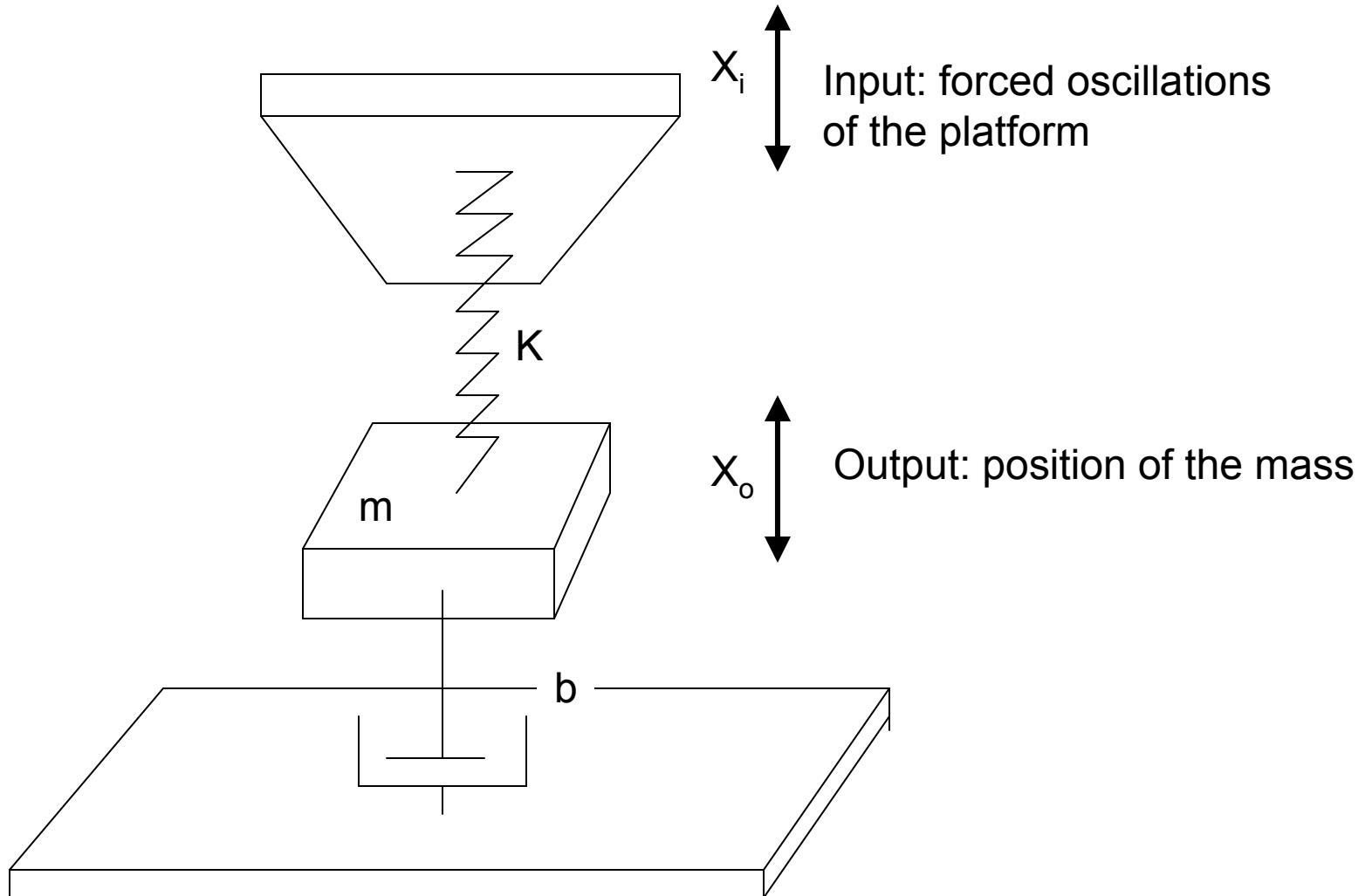
$$x_i(t) = x_i(f) \exp(j2\pi f t)$$



$$\frac{x_0}{x_i} = \frac{f_0^2}{f_0^2 - f^2 + j \frac{f f_0}{Q}};$$

# More about “displacement to displacement” transfer function

(from Saulson, page 61)



# Transfer functions

- Our specific problem:

the aim is to model the response of two test masses (horizontal displacement  $\rightarrow$  differential length) to the common horizontal noise motion of the ground.

Two transfer functions: stack and pendulum (different Q)

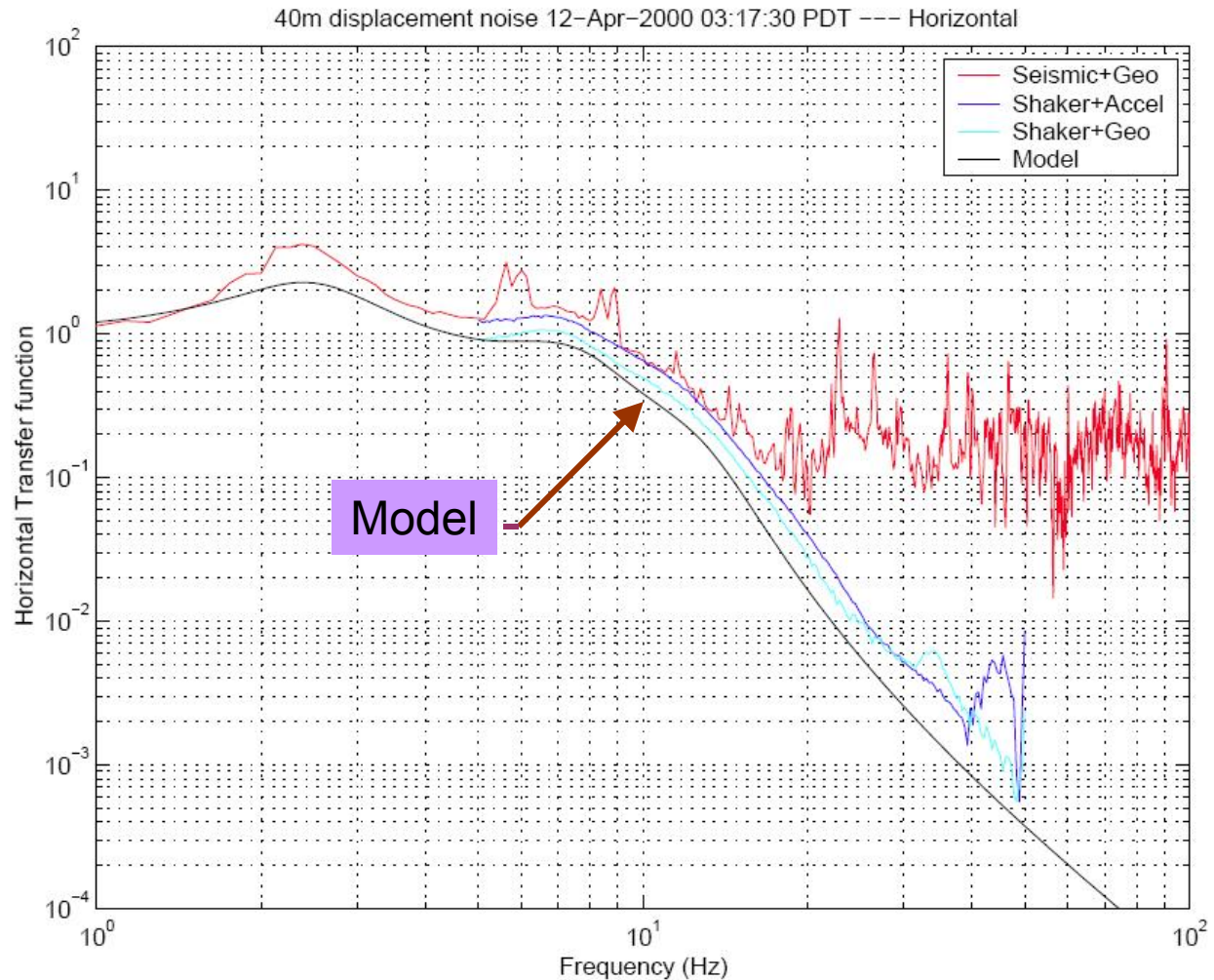
horizontal noise motion of the ground: from accelerometer,  
but calibration is necessary: from “counts/ $\sqrt{\text{Hz}}$ ”  $\rightarrow$  “m/ $\sqrt{\text{Hz}}$ ”

We study the horizontal displacement of the two test masses

# Transfer functions: stack

- See graph (plot) in Matlab from: LIGO-T000058-00-R, 05/11/2000, by D.Ugolini, S.Vass, A.Weinstein (page 11)

Three poles:  
2.5 Hz,  
7.5 Hz,  
13 Hz



# Transfer functions: stack

- I made a Matlab program that creates the transfer function of a very similar shape, that is  $\rightarrow$  with the same three poles  
(Matlab is new for me  $\rightarrow$  with Rana's help)
- Matlab commands:



```
zpk ( [], 2 * pi * [ p1, p2, p3 ], gain) ;
```

poles of the transfer function

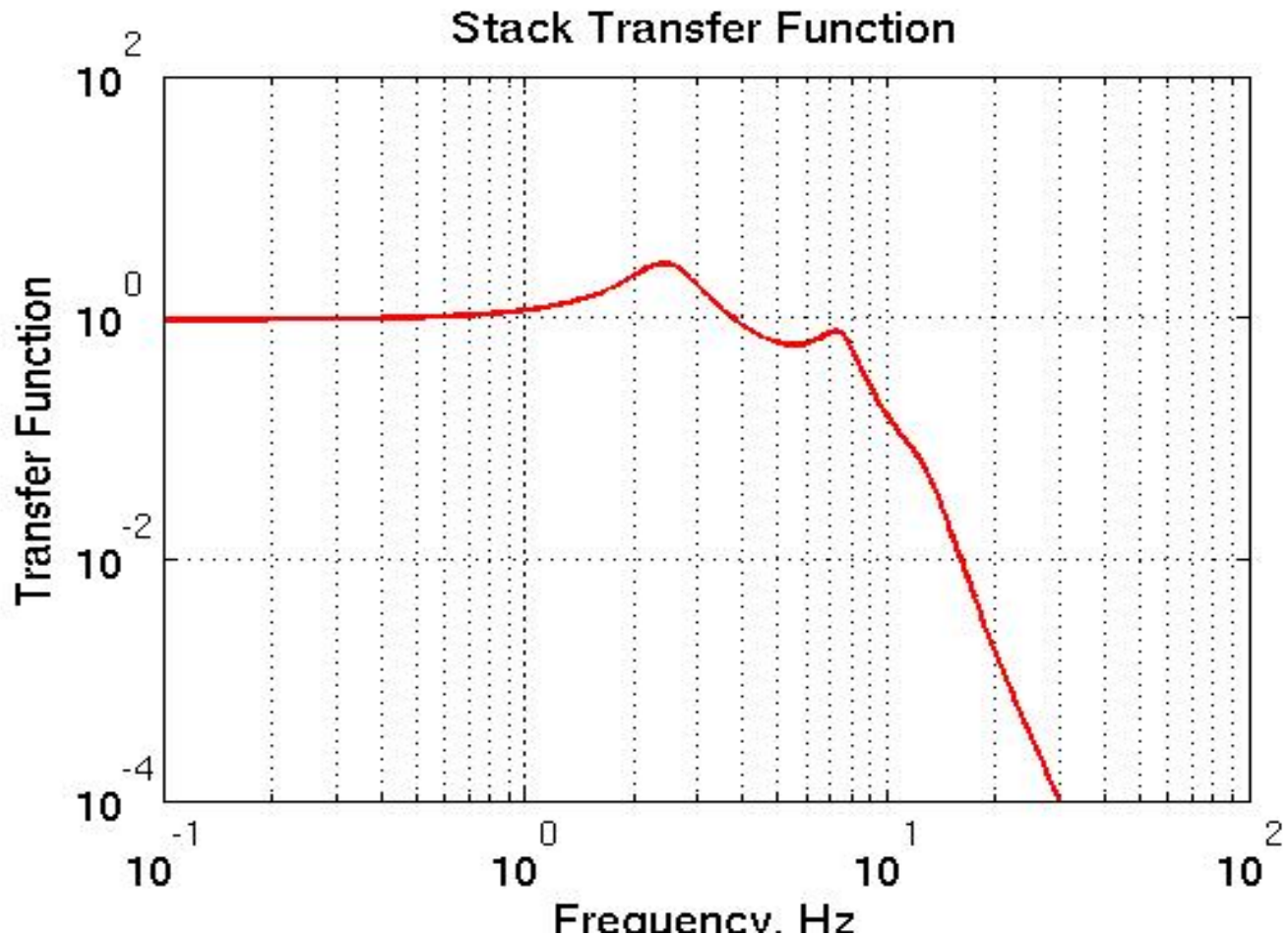
```
squeeze ( freqresp ( zpk( --// -- ), 2 * pi * frequency)
```

**➔** Analytical expression for the transfer function and plot

# Transfer functions: stack

- Analytical expression: 
$$\frac{3.5e9}{(s^2 - 6.283s + 246.6)(s^2 - 10.87s + 2221)(s^2 - 28.9s + 6667)}$$

- Plot:  
The same  
three  
poles

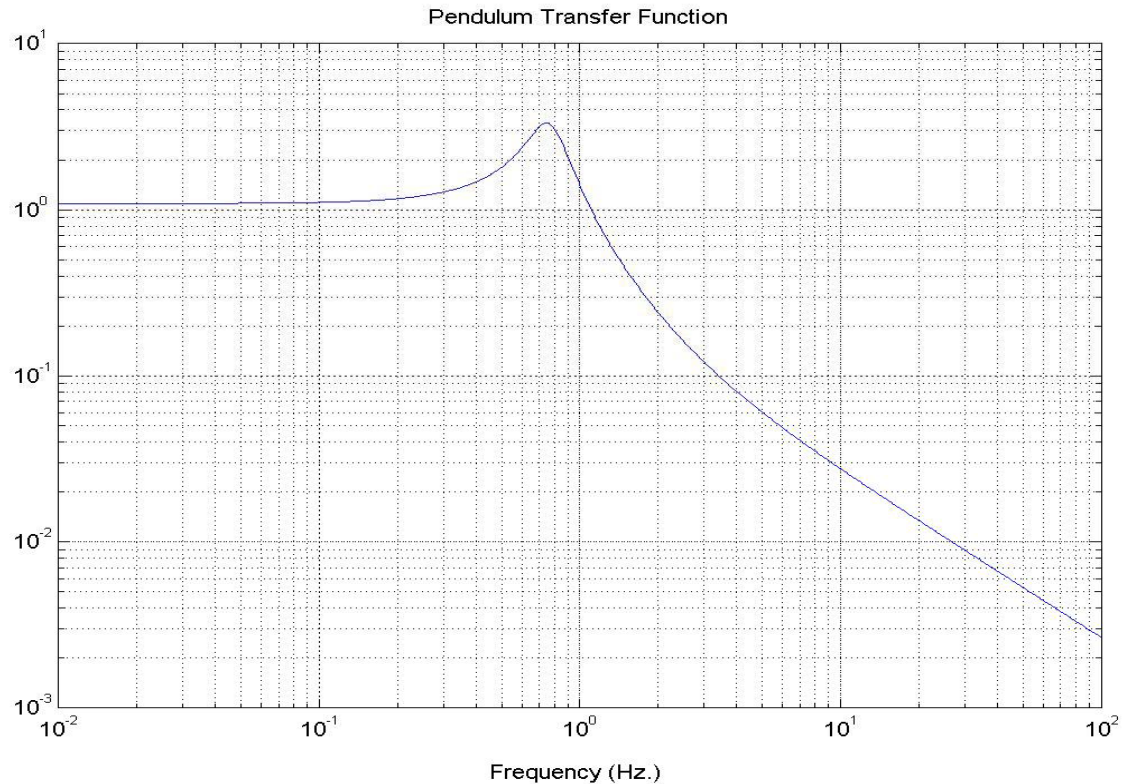


# Transfer functions: pendulum

- From Saulson, pages 61-62

$$\frac{x_o}{x_i} = \frac{f_o^2}{f_o^2 - f^2 + j \frac{f f_o}{Q}};$$

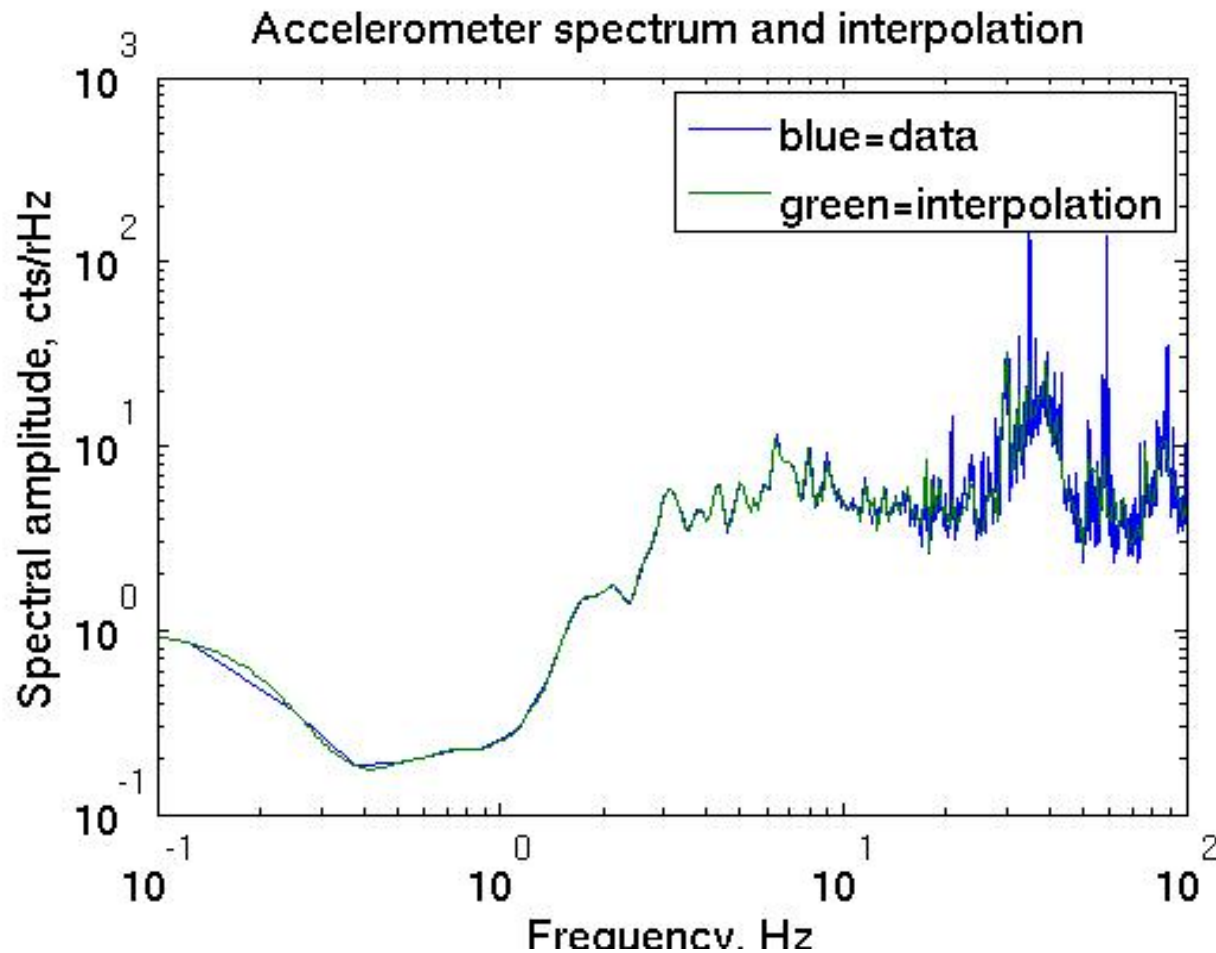
Use the following  
values:  
eigenfrequency 0.8Hz



# Input to our system: ground noise

- Accelerometer measurement.

Spectrum before calibration in “cts/ $\sqrt{\text{Hz}}$ ” units :



# Calibration: from “cts/ $\sqrt{\text{Hz}}$ ” to “m/ $\sqrt{\text{Hz}}$ ”

- **Step 1:**

Volt to g (acceleration) **gain conversion** for the accelerometer

- Wilcoxon calibration for the accelerometer 731A:
- Output 10 V/g for a gain of 1;
- Output 100 V/g for a gain of 10;
- Output 1000 V/g for a gain of 100.
- The current value of the switch for the gain: **gain = 10**

 We need to use coefficient **100 V/g**

- The accelerometer output signal has units ‘Volts per g’

$$\textit{Conversion} = 100 \frac{\text{V}}{\text{g}}$$

# Calibration

- **Step 2:** ADC voltage resolution (Volt / count)

The 40m has a digital control and readout system, therefore all data must be converted from an analog voltage to digital signal

- The ICS-110B 16-bit data acquisition board:
  - has a range of  $\pm 2$  Volt and a 16 bit resolution
- The conversion factor from counts back to volts is

$$V_R = \frac{\text{range}}{\text{resolution}} = \frac{4}{2^{16}} = 61.035 \frac{\mu V}{\text{count}}$$

Divide the ADC voltage resolution by the accelerometer gain conversion

$$\text{Conversion} = \frac{61.035 * 10^{-6} \frac{V}{\text{count}}}{100 \frac{V}{g}} = 59.8 * 10^{-7} \frac{m}{\text{count} * s^2}$$

# Calibration

- **Step 3:** Get position from acceleration

Each Fourier component obeys: 
$$y(t) = \frac{\ddot{y}(t)}{-\omega^2}$$

Then: The signal has units of “meter/count”:

$$\text{Conversion} = \frac{59.8 * 10^{-7} \frac{\text{meter}}{\text{count} * \text{s}^2}}{4\pi^2 f^2} = \frac{1.5 * 10^{-7} \text{meter}}{f^2 \text{count}}$$

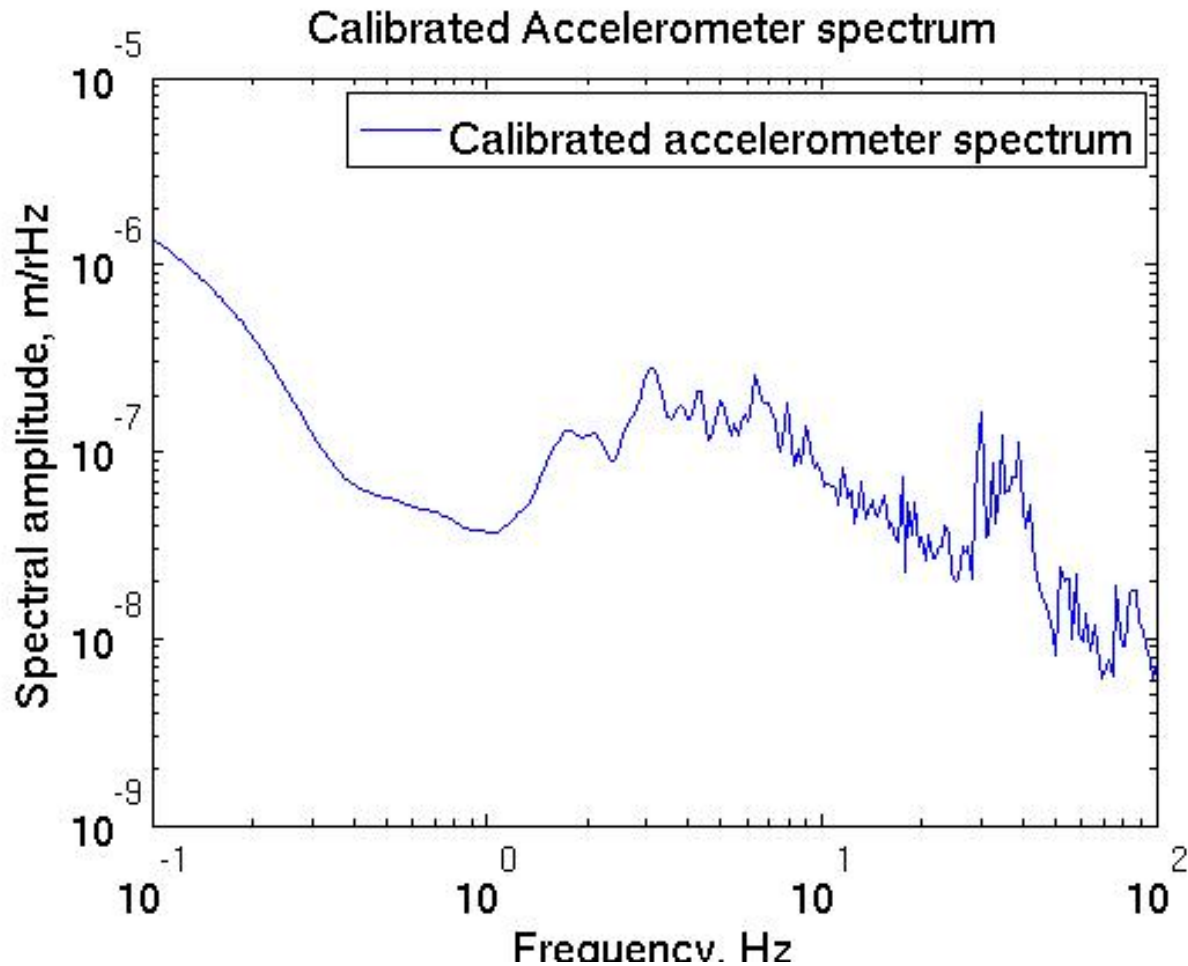
- **Step 4:** Get spectrum in units “meters/ $\sqrt{\text{Hz}}$ ” by multiplying

$$\frac{m}{\text{cts}} \frac{\text{cts}}{\sqrt{\text{Hz}}} = \frac{m}{\sqrt{\text{Hz}}}$$

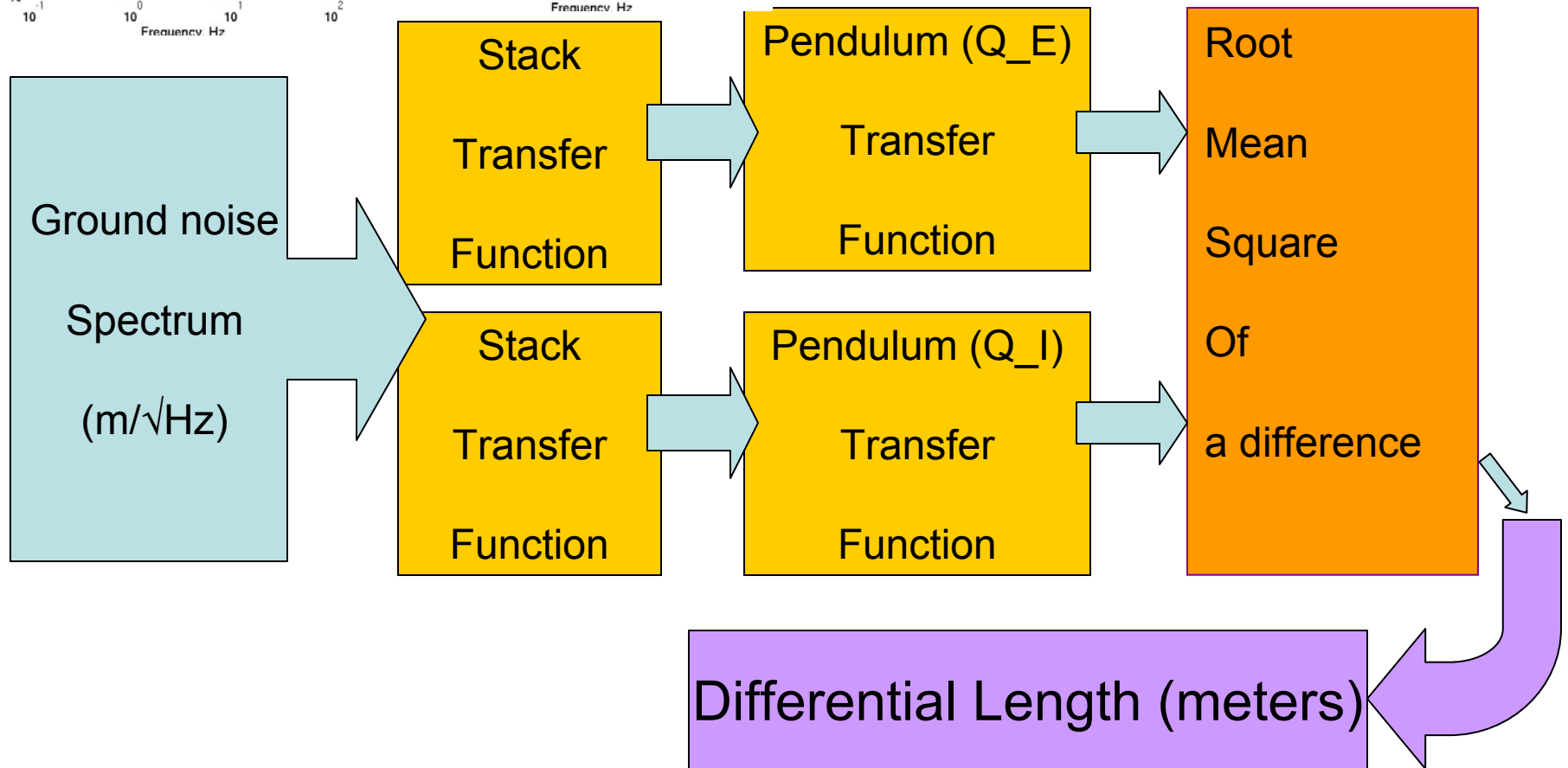
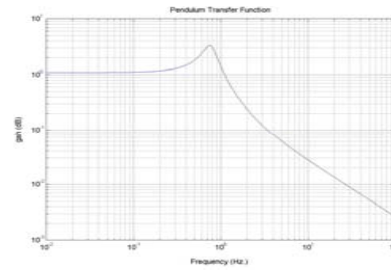
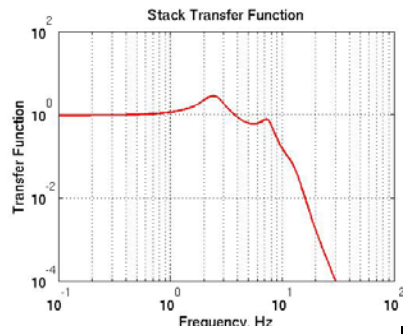
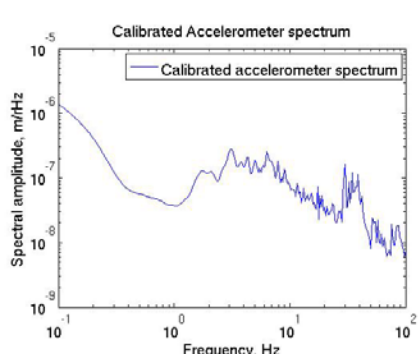
# Input to our system: ground noise

- Accelerometer measurement.

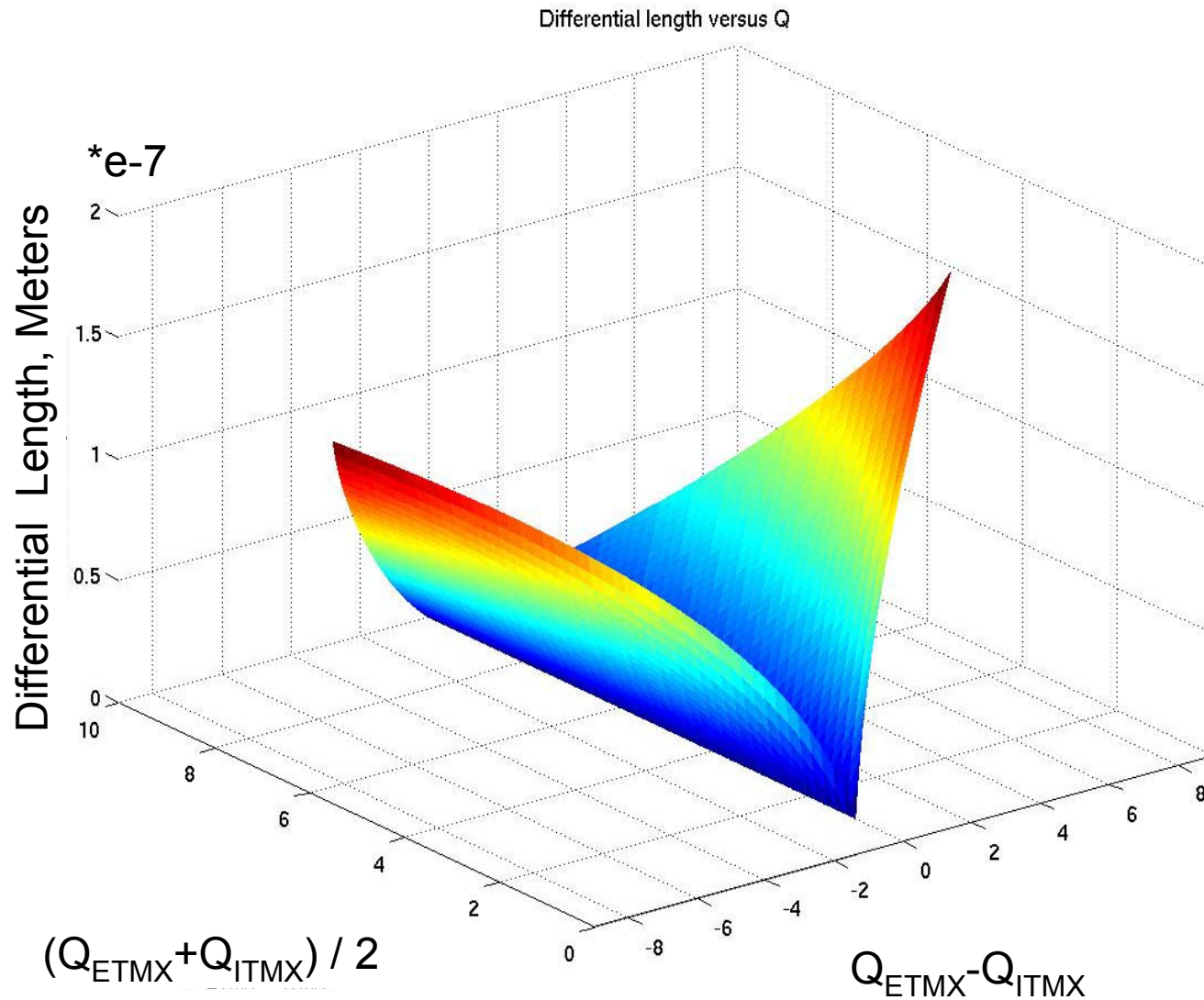
Spectrum after calibration in “meters/ $\sqrt{\text{Hz}}$ ” units :



# Calculation of the differential length

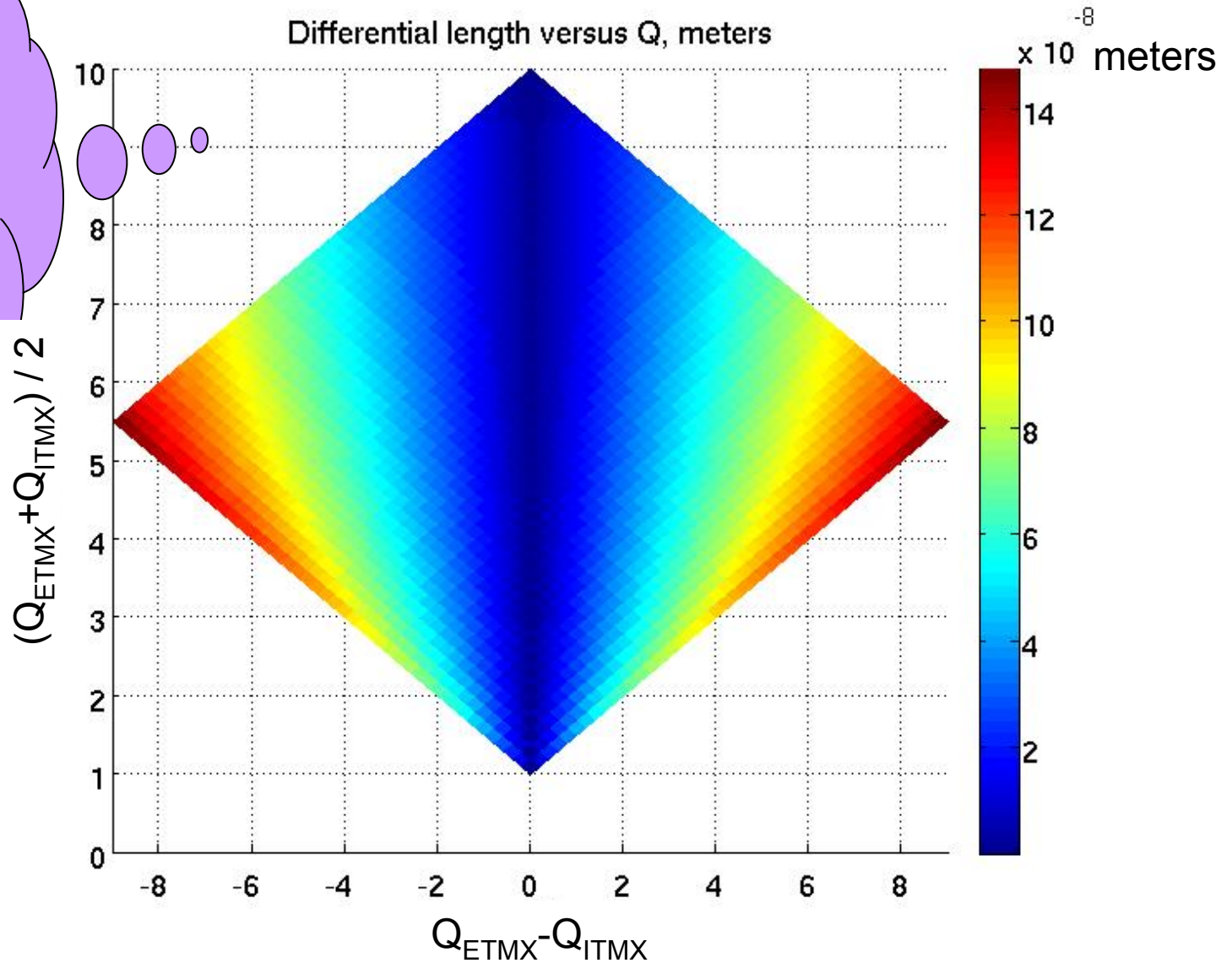


# Differential length as a function of $Q_E$ and $Q_I$



# Differential length as a function of $Q_E$ and $Q_I$

$1 \leq Q_E \leq 9$   
 $1 \leq Q_I \leq 9$



# Analysis and future plans

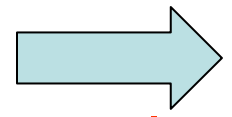
• **Minimum of differential length is for  $Q_I=Q_E$ .** It is easy to understand in our model. Remember, we had an underlying assumption:  $X_1=X_2=X_G$  (noise motion, absolutely rigid concrete slab)

If  $Q_I=Q_E$ , then all transfer function are the same, and we subtract two identical numbers from each other, getting zero.

True life is more complicated, and the “ideal” condition

$$X_1=X_2=X_G \text{ (noise motion, absolutely rigid concrete slab)}$$

is not fulfilled. Therefore, there might be some set of parameters  $Q_I$  and  $Q_E$  that the differential length has a minimum.



**measure the seismic noise at different points (move accelerometers) , and do not use condition  $X_1=X_2=X_G$ .**

# Applications of LIGO 40-m tripod



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# News from Moscow:

## the last chief of KGB has died

- **Former KGB chief Vladimir Kryuchkov dies at 83 MOSCOW** –

VLADIMIR KRYUCHKOV, the former KGB chief who spearheaded a failed coup against Soviet leader Mikhail Gorbachev, has died, officials said on Sunday. He was 83. Kryuchkov died on Friday in Moscow of an unspecified illness, said a spokesman for the Federal Security Service, the main KGB successor agency.

Kryuchkov owed his swift career rise to Soviet leader Yuri Andropov. He worked alongside Andropov when he served as the Soviet ambassador to Hungary, and oversaw a brutal suppression of anti-Communist uprising in Budapest in 1956.

When Andropov became KGB chief in 1967, he took Kryuchkov along and helped him rise through the ranks. In 1974, Kryuchkov was named chief of the KGB's First Main Directorate in charge of spying abroad. In 1988, Gorbachev appointed Kryuchkov as KGB chief. In Aug 1991, Kryuchkov joined other hard-line members of the Communist Party leadership who ousted Gorbachev and declared a nationwide state of emergency in an attempt to roll back liberal reforms. The coup collapsed after three days, and helped precipitate the collapse of the Soviet Union in Dec 1991. Kryuchkov and other coup plotters were jailed but later freed on an amnesty.

Kryuchkov's funeral took place on Tuesday.

