Input Mode Matching Telescope
40m Upgrade 2010

Jenne Driggers

February 9, 2010

1 Requirements

The new Input Mode Matching Telescope (MMT) should, within the space allowed in the 40m vacuum envelope, take the Input Mode Cleaner's beam waist of 1.66mm to the Arm Cavities' waist, which will be located on the ITMs and will have a spot size of 3.031mm. We would like the new MMT to be of a reasonable length to avoid the necessity of parabolic mirrors (which are used in the current/old Input MMT of length 140mm). We would prefer to use off-the-shelf mirrors, so radii of curvature options should be restricted to integer-meters, or half-integer-meters, either convex or concave. We must consider our ability to steer the beam coming from the Mode Cleaner into the arm cavities and so must consider where to place steering optics. Thus we also examine our ability to invert the control matrix for tilt and translation, depending on our choice of steering optics.

2 Design Analysis: Choosing the Best Solution

2.1 fminsearch.m to Find Set of Possible Solutions

This technique is borrowed from past generations of mode-matchers. We use Matlab's built-in fminsearch function to find the optimal mode matching solutions. We define 'optimal' as having the complex beam parameter q for the beam coming from the Mode Cleaner being identical to the q parameter of the Arm Cavity mode. This tells fminsearch that we want the waist spot size to match, and we want the location of the waists for each mode to match. We create the q parameter for the incoming beam by calculating the ABCD matrix for the optical chain that the beam must traverse, leaving several distances variable (the distance between the Mode Cleaner and the MMT, the length of the MMT, and the distance between the MMT and the PRM). A cartoon of the optical chain is shown in Figure 1, and the optical components used in the ABCD chain matrix are listed in Table 1.

We run the fminsearch optimization on every possible pair of MMT mirror radii of curvature that we can make using the optics for sale on the CVI website, for 2 inch Y1-coated (highly reflective for 1064nm) mirrors. Each optic set results in a solution from the fminsearch. We exclude all solutions for which the fminsearch did not converge (i.e. not a real solution), and all solutions which converged but have very poor match of waist spot size and waist location. We also exclude solutions which clearly will not fit in the 40m vacuum envelope (i.e. requiring 10's of meters for the telescope length).

For each solution which is not excluded, we examine how 'good' it is using a Monte Carlo simulation and by looking at its alignment capabilities.

2.2 Monte Carlo Simulation to Compare Solutions

One of the new features of this mode matching code is the use of a Monte Carlo simulation to estimate what our mode mismatch will be, given some set of random errors. We allow the radii of curvature of all

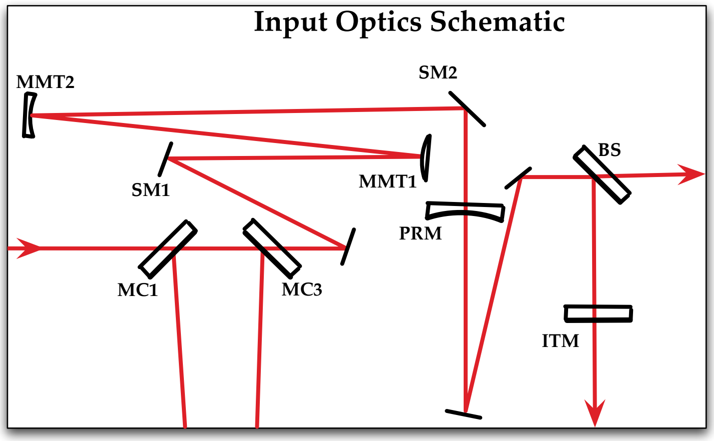

Figure 1: Schematic of Optical Chain relevant to the MMT. Not to scale.

Type	Component in Chain	Fixed or Variable
Dist	Waist of MC → MC3	Fixed
Thick Optic	MC3	Fixed
Dist	MC3 → MMT1	Variable
RoC	RoC of MMT1	Fixed, iterate over possible RoCs
Dist	Length of MMT	Variable
RoC	RoC of MMT2	Fixed, iterate over possible RoCs
Dist	MMT2 → PRM	Variable
Thick Optic	PRM	Fixed
Curved Dielectric Interface	RoC of PRM	Fixed
Dist	PRM → BS	Fixed
Thick Optic	BS	Fixed
Dist	BS → ITM	Fixed

Table 1: The Optical Chain relevant to the Input Mode Matching Telescope, waist of the Mode Cleaner to waist of the Arm Cavity

curved optics to vary within their specified tolerances (±0.5% for CVI 2 inch optics, used for the MMT mirrors, and ±5 meters for the PRM, which the beam must travel through). We allow the ideal positions of each individual optic to be misplaced by ±4mm. The actual error in the radii of curvature of the optics is out of our control, but the placement of the optics is. ±4mm seems very generous, and even with a very crowded table layout be able to place our mode matching optics to within 4mm. For each possible solution returned by the fminsearch, the Monte Carlo simulation takes all of our ideal parameters and adds normally distributed noise within the above mentioned error limits, then calculates the mode overlap integral between the beam coming from the Mode Cleaner, and the Arm Cavity mode. The mode mismatch, η, is given by Equation 1, where Z_{Rarm} is the Rayleigh Range of the Arm Cavity mode, and $Z_{R(xy)}$ is the Rayleigh Range of the X and Y transverse directions of the beam coming from the Mode Cleaner. The distance $z_{waist(XY)}$ is the difference in locations of the waist position between the Arm Cavity mode and the incoming beam, along the direction of propagation for both of the transverse directions. Including the X and Y directions separately allow us to incorporate the astigmatism resulting from non-normal incidence on the curved mode matching mirrors.

$$\eta = 4\sqrt{\frac{Z_{Rx}Z_{Ry}Z_{Rarm}^2}{\left(z_{waistX}^2 + (Z_{Rarm}+Z_{Rx})^2\right)\left(z_{waistY}^2 + (Z_{Rarm}+Z_{Ry})^2\right)}} \tag{1}$$

The simulation does 30,000 iterations per possible mode matching solution, and for each solution plots them in a histogram representing mode mismatch $(1-\eta)$. We are then able to look at the histograms for each possible mode matching solution and make an informed decision about which solution is most insensitive to errors in radii of curvature and optic placement. An example histogram (for our final design choice) is included in Figure 2.

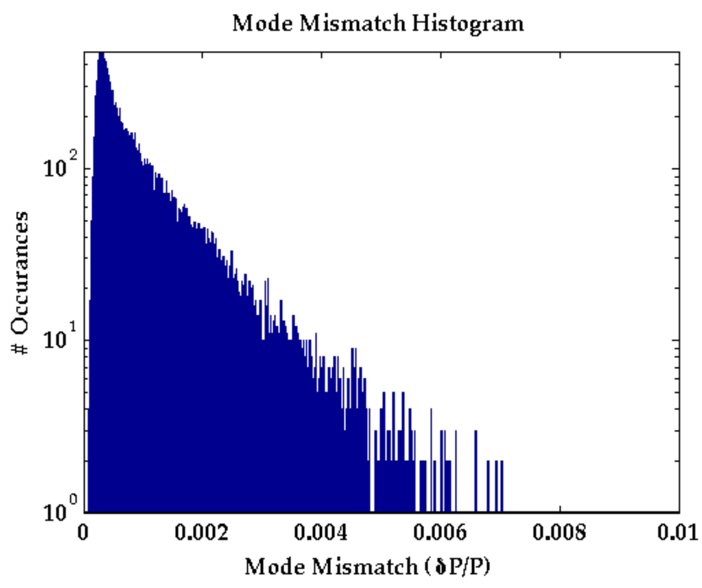

Figure 2: Mode Mismatch Histogram for our Final Design Choice

2.3 Tilt / Translation Orthogonality

In addition to examining the mode overlap, we look at the orthogonality of our tilt and translation alignment capabilities. We would like tilt and translation to be non-degenerate, so that we can invert a control matrix and actually move the steering mirrors in a way which isolates beam tilt and transverse translation.

To do this, we calculate the geometric optical chain ABCD matrix for the case where either one or the other Steering Mirror (SM1 and SM2 in Figure 1) are tilted by some angle. Since we use the geometric picture here (versus Gaussian optics), we get an output vector of beam translation and beam tilt angle (one vector for each Steering Mirror's tilt). We normalize these vectors by appropriate values: the translation components are normalized by the Arm Cavity's waist spot size,ω_{want}, and the beam angle components are normalized by the divergence angle of the Arm Cavity mode: $\gamma = \lambda / (\pi * \omega_{want})$. We then take the difference in arctangents of (normalized translation / normalized angle), and define that to be our orthogonality angle.

We plot a color surface plot, shown in Figure 3, what the alignment angle could be if we moved one or the other flat steering mirrors. The take-away message from this plot is primarily that we will not gain an incredible amount of orthogonality by moving one of the mirrors by a small amount. Thus, we should give more weight to finding a set of optic placements which fit nicely on the tables and give us good mode matching overlap. If we saw, however, that moving one of the steering mirrors by a small amount could greatly increase our tilt / translation orthogonality angle, it would be worth considering moving the optics around.

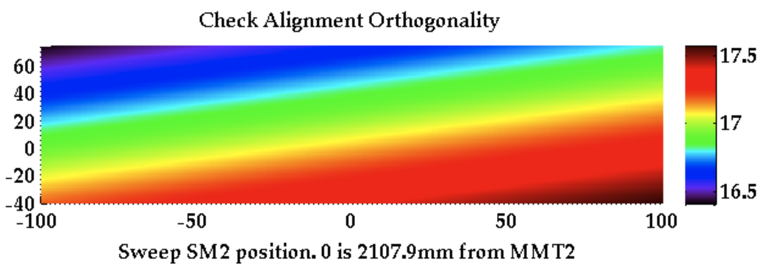

Figure 3: The tilt / translation orthogonality angle (color) as a function of moving the 2 steering mirrors (X and Y axes)

For the mode matching solution chosen for the 40m Upgrade MMT, we get an orthogonality angle of 17 degrees. While this is not particularly orthogonal, it is enough that we should be able to invert the control matrix. Also, it is likely much better than the old/current 40m steering capability, since the steering mirrors are very close to one another.

2.4 Beam Profile Plots

As a final check to ensure that the final waist resulting from our mode matching solution is near the same place as the ITM (since the ITMs are flat mirrors, the waist of the Arm Cavity will be at the ITMs), we plot the beam radius versus propagation distance, between the Mode Cleaner and into the arms. The vertical line on the plot indicates the location of the ITM (average distance since the X and Y ITMs are slightly different distances from the Beam Splitter). The beam profile for our final chosen solution is shown in Figure 4.

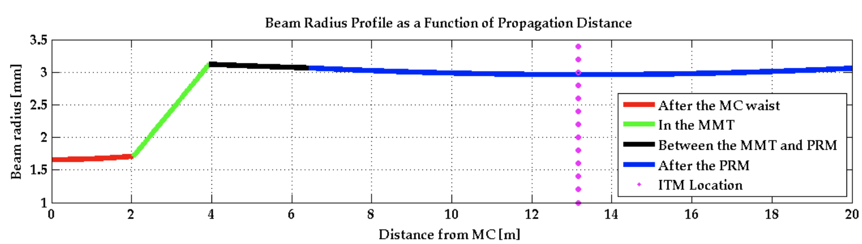

Figure 4: Beam Profile Plot for our Final Design Choice

3 Table Layout Constraints

One of the most complicated parts of choosing a mode matching solution was finding a solution which fits in the 40m vacuum envelope, fits on the tables we have, and doesn't interfere with other beams in our very crowded environment. In the end, we've chosen a solution which spans 3 optical tables / chambers: the first steering mirror will be on the IOO table, MMT mirror 1 will be on the BS table, MMT mirror 2 will be on the Output Optics table, and the final steering mirror will also be on the BS table. This final layout is shown in Figure 5. Compare Figure 5 with Figure 1 for Mode Matching optics.

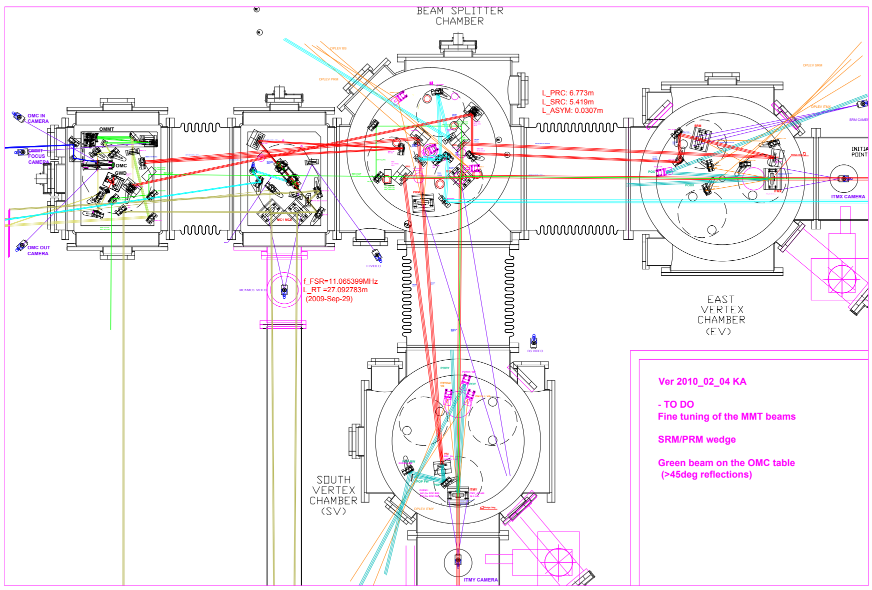

Figure 5: Table Layout for Input Mode Matching Telescope and other IFO optics as of 3 Feb 2010

Once we had distance parameters which we knew would fit on the tables, we reexamined our possible mode matching solutions and chose the one which looked most feasible. We then placed the mirrors in the optical layout diagram, and measured the final distances. These distances, with the radii of curvature of the mode matching optics were put back into the mode matching code, to see what the final histogram and tilt / translation angle would look like. We also plot the mode match overlap versus optic position for the distances relating to the telescope (sliding the telescope around, or changing the length of the telescope along with the distances immediately before or after the telescope). We find the maximum possible mode match overlap (presumably a small correction to the distances we have used in our optical layout, if those distances were informed by the optimal solution found using the fminsearch), and correct the placement of the optics to match this maximum. It turns out for the solution that we had chosen, optimum was to move MMT mirror 1 by 14mm, so we were already very close to where we wanted to be.

4 Final Design Choice

The final parameters for the mode matching telescope are:

We plot the mode match overlap η versus optic position for several different cases of moving / changing the telescope, shown in Figure 6. We see in the lower 3 panels of the figure that after the fine tuning mentioned in Section 3 that we have the optimal possible mode match overlap, and that we are able to maintain above 99.9% if we move either of the MMT optics by as much as ∼2cm.

We can also look at the histograms, for different values of the optic position error, and find the fraction of cases where the mode match overlap is worse than 99.9% to the total number of cases shown in the

Name in .m Code	Start Optic	End Optic	Distance [mm]
d2a	MC3	SM1	884.0
d2b	SM1	MMT1	1044.2
d3 (Telescope Length)	MMT1	MMT2	1876.0
d4a	MMT2	SM2	2007.9
d4b	SM2	PRM	495.6
d5	PRM	BS	4443.3
d6	BS	avg ITM	2273.8

Table 2: Final choice of distances between optics in new Input Mode Matching Telescope

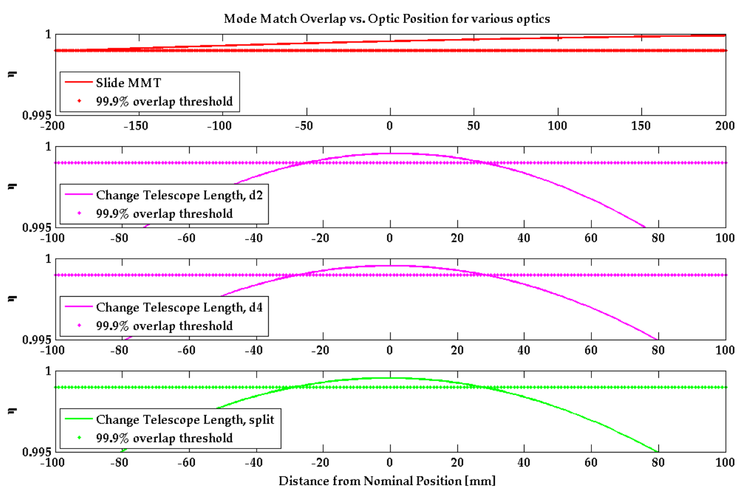

Figure 6: Mode Match Overlap versus moving an optic. The first panel is sliding the mode matching telescope around. Panel 2 is moving MMT1 (neg is closer to the IOO table). Panel 3 is moving MMT2 (with negative again moving closer to the IOO table. Panel 4 is moving MMT1 and MMT2 equally, with the same sign conventions as the other panels.

histogram. We do this twice, once with all of the other errors (that is, the errors in the radii of curvature of the optics) set to zero, and once with the errors as they are in Section 2.2. When the only errors allowed are the distances, we find the following:

- Position error: $\sigma = 1$mm. Fraction worse than $\eta = 99.9\%$: 0%
- Position error: $\sigma = 5$mm. Fraction worse than $\eta = 99.9\%$: 0%
- Position error: $\sigma = 10$mm. Fraction worse than $\eta = 99.9\%$: 1%
- Position error: $\sigma = 15$mm. Fraction worse than $\eta = 99.9\%$: 8%
- Position error: $\sigma = 20$mm. Fraction worse than $\eta = 99.9\%$: 18%
- Position error: $\sigma = 50$mm. Fraction worse than $\eta = 99.9\%$: 60%
- Position error: $\sigma = 100$mm. Fraction worse than $\eta = 99.9\%$: 81%

When we include the errors in the radii of curvature, we find:

- Position error: $\sigma = 1$mm. Fraction worse than $\eta = 99.9\%$: 26%
- Position error: $\sigma = 5$mm. Fraction worse than $\eta = 99.9\%$: 27%
- Position error: $\sigma = 10$mm. Fraction worse than $\eta = 99.9\%$: 30%
- Position error: $\sigma = 15$mm. Fraction worse than $\eta = 99.9\%$: 35%
- Position error: $\sigma = 20$mm. Fraction worse than $\eta = 99.9\%$: 40%
- Position error: $\sigma = 50$mm. Fraction worse than $\eta = 99.9\%$: 66%
- Position error: $\sigma = 100$mm. Fraction worse than $\eta = 99.9\%$: 83%

When we look at a single value of σ for the distance error, but only allow one distance at a time to vary (holding the others at zero error, the distances are defined as in Table 2), we notice very clearly that the dominant distance which makes the mode match overlap decrease is changing the length of the mode matching telescope.

We conclude that we may misplace any optic by as much as 10mm, and still have a high probability of having excellent mode matching. We expect that we should be able, by using reasonable rulers, to place the optics within $\sim$2mm or so, so we should have no trouble getting very good mode matching into the interferometer.