LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Calculating the strength of GigE Phase Camera

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1 Overview

This document is intended to determine the strength of a signal detected by a CCD using short exposure times and a reference beam of order a kilohertz away from the frequency being examined.

2 Calculations

We begin with the carrier light with amplitude A_c and sidebands with amplitude A_{sb+} and A_{sb-}. These are at frequencies ω_c,ω_{sb+} respectively. We then beat this with a reference beam with amplitude A_r and frequency ω_r. There is an overall phase associated with each, but we'll assume for now that its zero to simplify calculations. It doesn't qualitatively change the results. We also assume the amplitudes are purely real.

$$\begin{aligned} Power = \int_0^T |A_c|^2 + |A_{sb+}|^2 + |A_r|^2 + |A_{sb-}|^2 + A_c A_{sb+} e^{-i(\omega_{sb+}-\omega_c)t} + A_c A_{sb+} e^{-i(\omega_c-\omega_{sb+})t} \\ + A_c A_{sb-} e^{-i(\omega_{sb-}-\omega_c)t} + A_c A_{sb-} e^{-i(\omega_c-\omega_{sb-})t} \\ + A_r A_{sb+} e^{-i(\omega_{sb+}-\omega_r)t} + A_r A_{sb+} e^{-i(\omega_r-\omega_{sb+})t} \\ + A_r A_{sb-} e^{-i(\omega_{sb-}-\omega_r)t} + A_r A_{sb-} e^{-i(\omega_r-\omega_{sb-})t} \\ + A_c A_r e^{-i(\omega_r-\omega_c)t} + A_c A_r e^{-i(\omega_c-\omega_r)t} \end{aligned} \quad (1)$$

For an integration time T which is of order 1 microsecond and assuming $\omega_c - \omega_{sb+}$, $\omega_c - \omega_{sb-}$, $\omega_c - \omega_r$ and $\omega_r - \omega_{sb-}$ are all of order 10^6 cycles per second or more, then the time varying components for those frequencies can be neglected. This is because they're contribution falls off with the integration time T relative to all the other components.

This leaves us just with

$$Power = \int_0^T |A_c|^2 + |A_{sb+}|^2 + |A_{sb-}|^2 + |A_r|^2 + A_r A_{sb+} e^{-i(\omega_{sb+}-\omega_r)t} + A_r A_{sb+} e^{-i(\omega_r-\omega_{sb+})t} \quad (2)$$

We can use $2\cos(x) = e^{-ix} + e^{-ix}$ to write

$$Power = \int_0^T |A_c|^2 + |A_{sb+}|^2 + |A_{sb+}|^2 + |A_r|^2 + A_r A_{sb+} 2\cos((\omega_r - \omega_{sb+})t) \quad (3)$$

$$Power = \left(|A_c|^2 + |A_{sb+}|^2 + |A_{sb+}|^2 + |A_r|^2\right) T + \frac{2A_r A_{sb+} sin((\omega_r - \omega_{sb+})T)}{\omega_r - \omega_{sb+}} \quad (4)$$

In the case where $(\omega_r - \omega_{sb+})T << 1$, we can approximate the above as

$$Power = \left(|A_c|^2 + |A_{sb+}|^2 + |A_{sb+}|^2 + |A_r|^2 + 2A_rA_{sb+}\right)T \tag{5}$$

As long as $2A_rA_{sb+}$ is a significant fraction of the total sum, we should be able to detect its variation. In fact the above is a maximum power that will be detected, while the minimum power would be

$$Power = \left(|A_c|^2 + |A_{sb+}|^2 + |A_{sb+}|^2 + |A_r|^2 - 2A_rA_{sb+}\right)T \tag{6}$$

in the case where the light is 180 degrees out of phase with the first (we had been neglecting phase up to this point, but it just adds an abitrary phase factor to the time varation in eq. 4. Subtracting these two equations yields $4A_rA_{sb+}T$. As long as we're not saturated by the high point in power (eq. 5), we have sufficient resolution to detect a difference of $4A_rA_{sb+}T$, and $(\omega_r - \omega_{sb+})T << 1$ then using the CCD as a phase camera should work.