LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Update and Additions to the 40 m Noise Budget, First Progress Report

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1 Overview of Project

The Laser Interferometer Gravitational Wave Observatory (LIGO) is one of the most ambitious projects in modern physics. Originally a joint effort between Caltech and MIT, LIGO has blossomed into approximately 40 institutions participating to achieve the following goals. [5]

1. Verify by direct observation the existence of gravitational waves as predicted by general relativity.
2. Use the world's first gravitational observatory to make unique observations of the universe's most dense and least understood phenomena.

To achieve these goals LIGO employs three heavily modified Michelson Interferometers as gravitational wave observatories. The first observatory has an arm-length of 4 km and is located in Hanford, Washington along with the second observatory with a 2 km arm-length. The third observatory is located in Livingston, Louisiana and has an arm-length of 4 km. Multiple observatories located far from each other allows LIGO analysts to mitigate local effects by demanding coincident detection of a gravitational wave. Furthermore, with separated observatories, analysts may triangulate the source of an incoming signal. General relativity predicts that sources such as inspiraling neutron stars and certain types of stellar collapse will emit strong, polorized gravitational waves that will radiate outward through space at the speed of light. When such a gravitational wave passes through the dectector, one arm of the interferometer will lengthen while the other will contract. The 4 km interferometers can detect a change in arm-length on the order of 10^{-18}. Because the volume of space probed is proportional to the cube of the strain sensitivity [7], minimization of environmental and instrumental noise sources becomes paramount.

The level of precision required for the observatories, roughly a thousandth the diameter of a proton, demands careful catalogue and control of noise sources. An accurate catalogue of noise sources and their effect on the interferometer is necessary to determine what is limiting the accuracy of the observatory [1]. This catalogue is a noise budget.

2 Previous Work on 40m Noise Budget

Previous work to construct a noise budget for the 40m facility has centered around modifying existing methods for creating an automated noise budget. The sites at Livingston, Louisiana and at Hanford, Washington both use a set of MATLAB scripts to regularly create noise budgets. The goal is to modify that code to work at the 40 m site. These modifications are non-trivial since new transfer functions must be measured for each noise source and new sources specific to the 40 m site must also be added to the scripts while preserving cross-site functionality. Previous SURF students Ryan Kinney and David Malling both worked on the 40 m noise budget. Kinney measured the transfer for the seismic noise source and added it to the working noise budget. I intend to build on their work by adjusting the seismic noise budget to work with the current 40 m configuration and adding PRC, SRC, and OSEM noise sources.

3 Noise Budget Development

A block diagram of the current noise budget script is shown below. The procedure begins with the activation of getNoiseBudget.m which together with runmeas.m sets file paths specific to each site and calls the script NoiseBudget.m. NoiseBudget.m updates the preloaded parameter files with updatepardata.m. Then NoiseBudget.m calculates the unitary gain frequency with getUGF.m and takes data from each of the source channels with files such as getSeismic.m. The get files take data and with a known transfer function calculates the noise contribution. NoiseBudget.m then plots and saves the noise budget.

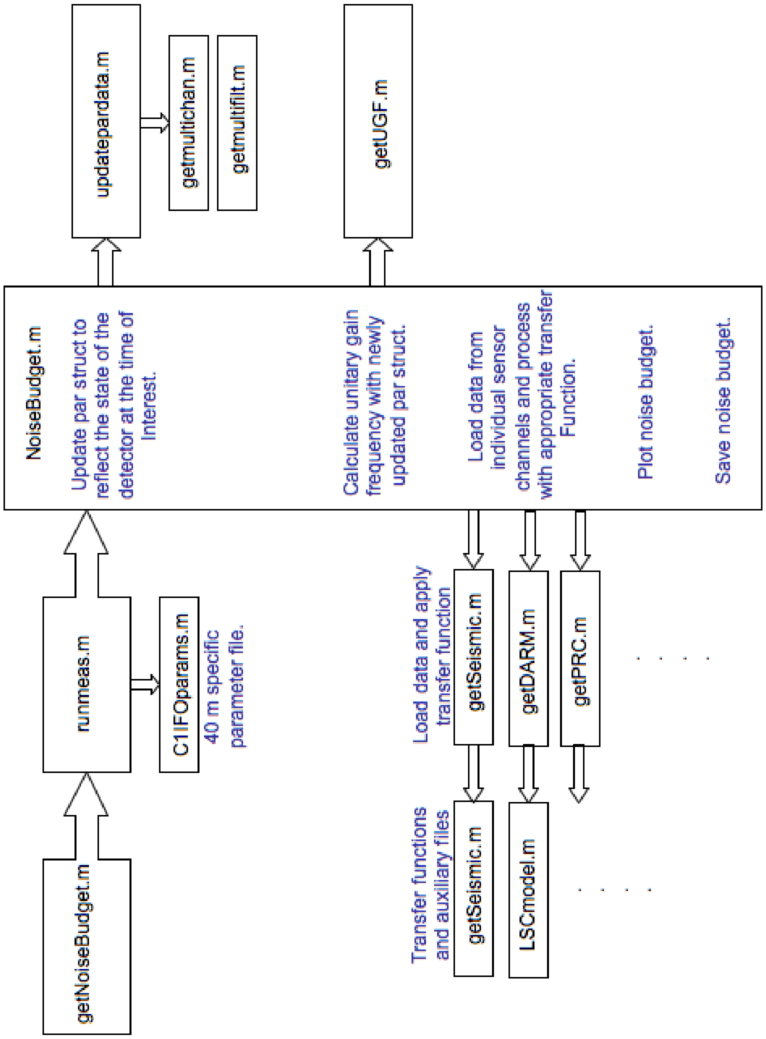


Figure 1: Noise Budget Script Block Diagram

3.1 Seismic Sources

Among the primary gravitational wave sources LIGO hopes to detect are inspiring neutron stars (or black holes) and primordial gravitational wave background. It is thought that inspirals will emit gravitational waves in the range of a few hundred hertz or lower for a brief period of time before coalescence and furthermore that the gravitational background may be on the order of a 100 hertz. [10]. Therefore, it is extremely important for LIGO to achieve maximum sensitivity in this frequency band.

At frequencies lower than 100 hertz, the main source of noise is seismic motion at the sites. Ground motion is translated into optical motion in the arm cavities which is translated to positive signal at the dark port. Because a passing gravitational wave it also detected by a positive signal at the dark port, constant stochastic seismic motion threatens to create noise that could swallow signals in this frequency band.

To damp optical motion due to seismic activity, all major optics are suspended from pendulums. These pendulums are in turn bolted to the top of a series of four spring and plate passive seismic isolators. The spring and plate system is referred to as the seismic isolation stack. The pendulum is designed to have a resonant frequency of .75 hertz [11], well below the frequencies which LIGO expects to be able to detect gravitational waves. The same is true for each of the spring/mass systems comprising the stack. Far from resonance, these elements are approximately simple harmonic oscillators and so the frequency dependent amplitude of their oscillations $A(\omega) \propto \omega^{-2}$. Therefore the total transduced amplitude A_{total} is the product of the amplitude attenuations of the each element, $A_{total} \propto \omega^{-8}$ [12]. As an aside, the LIGO facility in Livingston, Louisiana also employs and active seismic isolation system due to its proximity to a set of railroad tracks. The system consists of accelerometers which are connected in a feed back loop to actuators located on the stack. When the accelerometers detect ground movement, the actuators attempt to compensate. Advanced LIGO will use active seismic isolation at all sites.

3.1.1 Seismic Noise Calculation

As explained above, the noise budget script employs the the function getSeismic.m to calculate the contribution of seismic sources to noise in the DARM signal. Two 3-axis Wilcoxon 731A accelerometers are located near the mode cleaner with x- and y-axis accelerometers parallel to the ground. The accelerometers are connected to control room computers which read each channel in a dimensionless unit counts. The function getSeismic.m samples these channels for a given period of time, converting counts to acceleration using the following formulation [9]:

$$\frac{4 \text{ volt}}{16 \text{ bit resolution}} = \frac{4 \text{ volt}}{2^{16} \text{ bits}} = 61.035 \frac{\mu v}{count}$$

The accelerometers transduce acceleration along the axis into output voltage using the following conversion factor

$$\frac{100\text{V}}{9.8\frac{\text{m}}{\text{s}^2}}$$

so that the total conversion factor is $5.9824 * 10^{-6}$m cts s^{-2}.

The ground is assumed to be a simple harmonic oscillator oscillating at frequency ω so that to first approximation the ground displacement magnitude is equal to ground acceleration divided by ω^2. Given the ground acceleration, transfer functions modeling the stack and the pendulum are used to convert ground displacement into optical displacement

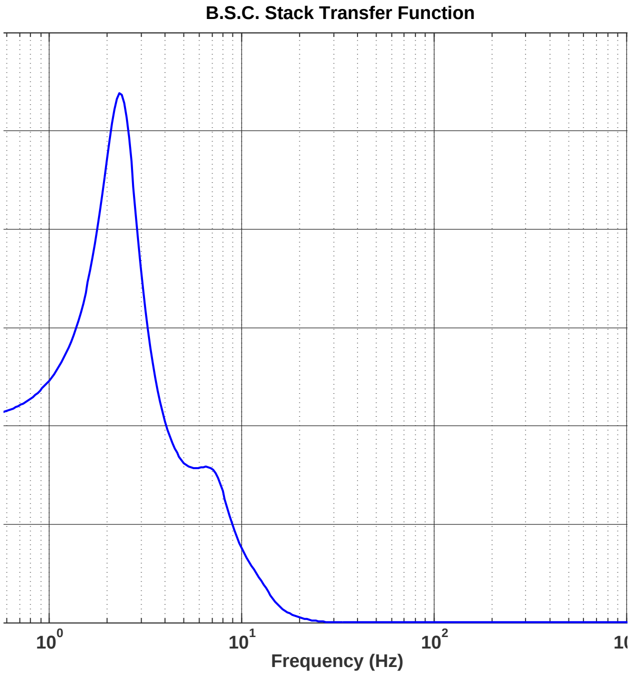


Figure 2: Stack Transfer Function for the Beam Splitter Chamber

3.2 DARM noise

The differential arm length channel (DARM) C1:LSC-DARM_ERR measures the intensity of light being detected at the dark port. This information is then related to the phase difference between light traveling down each arm from which the presence of a gravitational wave may be inferred. Under normal operation there is no gravitational wave so that there should be no light at the dark port. This is especially true for the 40 m facility since the comparatively short length of the arms implies that the interferometer lacks the ability to actually detect gravitational waves. Therefore, any signal from C1:LSC-DARM_ERR is considered noise and moreover this noise is a direct measure of LIGO's ability to detect gravitational waves. Models are applied to other noise sources specifically to estimate the affect of each source on this channel. The estimated total predicted noise is then compared to the actual noise in the C1:LSC-DARM_ERR channel to test the possibility of inaccurate models or unaccounted for noise sources.

4 Current State of Noise Budget

The current noise budget is shown at the end of the paper. I have re-written getSeismic.m and it's ancillary function SeismicNoise40.m to reflect the current state of the 40 m facility and to be clearer and more easily modifiable. To measure the seismic magnetic fields affecting the beam splitter, I have calibrated a Bartington 3-Axis Flux-gate Magnetometer and installed it near the beam splitter chamber. Appendix B details the calibration procedure. I am currently working to revise the matlab scripts which incorporate DARM, MICH, and PRC noise.

References

[1] R. Adhikari.

[2] J. Garofoli.

[3] D. Malling: *Development of a Prototype for the LIGO 40 m. Prototype*, SURF Paper, 2006.

[4] Unknown: *Shot Noise*, http://en.wikipedia.org/wiki/Shot_noise.

[5] Unknown: *LIGO*, http://en.wikipedia.org/wiki/LIGO.

[6] Unknown: *LIGO Fact Sheet*, http://www.ligo.caltech.edu/LIGO_web/about/factsheet.html.

[7] LIGO Science Collaboration: *LIGO: The Laser-Interferometer Gravitational-Wave Observatory*, http://www.arXiv.org, 2007.

[8] R. Kinney: *Development of Noise Budget for the LIGO 40m IFO*, SURF Paper, 2005.

[9] R. Kinney: *Noise Budget Development for the 40 Meter Prototype*, SURF Presentation, 2005.

[10] J. Giame et al. : *A passive vibration isolation stack for LIGO: Design, modeling, and testing*, Review of Scientific Instruments, January 1996.

[11] R. Adhikari: *Sensitivity and Noise Analysis of the 4 km Laser Interferometric Gravitational Wave Antennae*, Thesis, July 2004.

[12] F. Crawford: *Waves : Berkley Physics Course 3*. unknown copyright.

A Calibration of the LIGO 40m Bartington Magnetometer

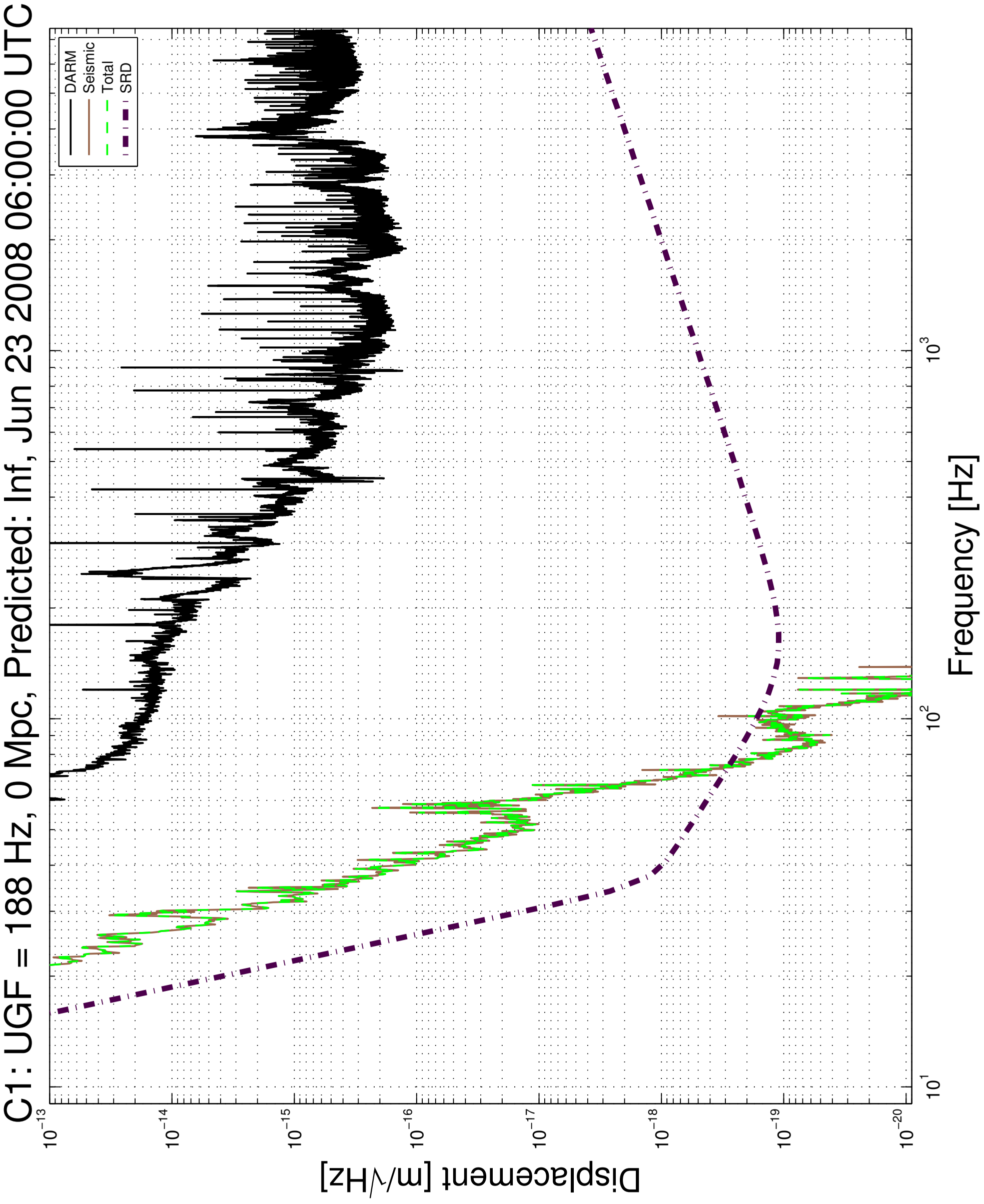


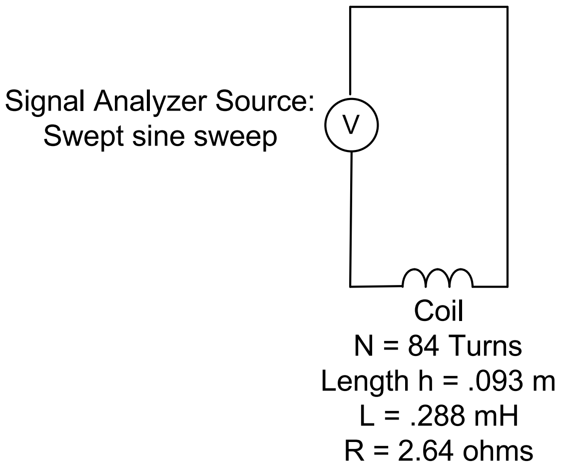
Figure 3: 40 m Noise Budget

A.1 Overview

The Bartington MAG03MC Three Axis Magnetometer is designed to measure slow changing magnetic fields of strength ±100μT (with 12 volt power supply). The magnetometer is necessary to measure the static magnetic fields around the test masses which are currently an unaccounted for source of noise. Because the magnetometer was purchased in 1992 and has been out of use for several years, it was necessary to calibrate the magnetometer.

A.2 Set - Up

To calibrate the three perpendicular axis, two different experimental set-ups were employed. Both set-up s employed the circuit displayed below.



The spectrum analyzer performed a swept sine transfer function measurement by outputtting a voltage sine wave of specified amplitude and varying frequencies into the circuit shown above. The resulting oscillating magnetic field produced an oscillating voltage signal from the magnetometer probe. This signal acted as the input for the spectrum analyzer.

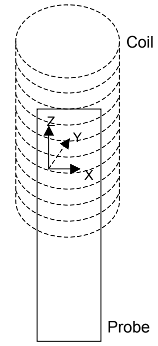
To calibrate the z-axis of the magnetometer (the axis parallel with the probe length), the probe was inserted into the middle of the coil such that the z-axis flux-gate magnetometer was at the center of the coil. To calibrate the x- and y-axis flux-gate magnetometers, the coil was moved such that the the the end field of the coil was perpendicular to the axis being calibrated as shown in the figure below.

A.3 Theory

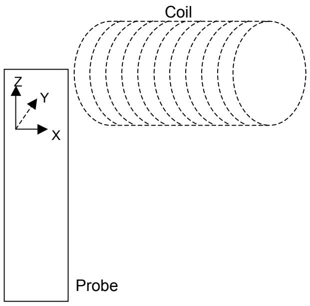
Let V_0 be the rss (root sum of squares) voltage read at the input channel of the spectrum analyzer. We then assume based on the magnetometer manual that

$$\begin{aligned} V_o &= \alpha B \\ &= \alpha(B_{bk} + B_{coil}) \end{aligned}$$

where B_{bk} is the background magnetic field and B_{coil} is the magnetic field resulting from the coil. We know from basic physics courses that if we assume that the coil is sufficiently long



Z-axis Calibration



X- and Y-axis Calibration

in comparison to it's width then the field B_{coil} is parallel to the direction of the solenoid with the magnitude

$$B_{coil} = \mu_0 \frac{NI}{h}$$

where

$$I = \frac{V}{Z} = \frac{V_i cos(\omega t)}{Z}$$

where the impedance Z is given by

$$Z = R + j\omega L$$

so that

$$I = \frac{V_i cos(\omega t)}{R + j\omega L}$$

By using a spectrum analyzer in swept sine mode we may eliminate the background magnetic field from consideration since only the magnetic field produced by the coil will oscillate with the same frequency as the input voltage. The transfer function $\frac{V_o}{V_I}$ will be given by

$$\begin{aligned} \frac{V_o}{V_i} &= \alpha B_{coil} \\ &= \mu_0 \frac{\alpha N}{h} \frac{cos(\omega t)}{R + j\omega L} \end{aligned}$$

Therefore, the magnitude of the transfer function is

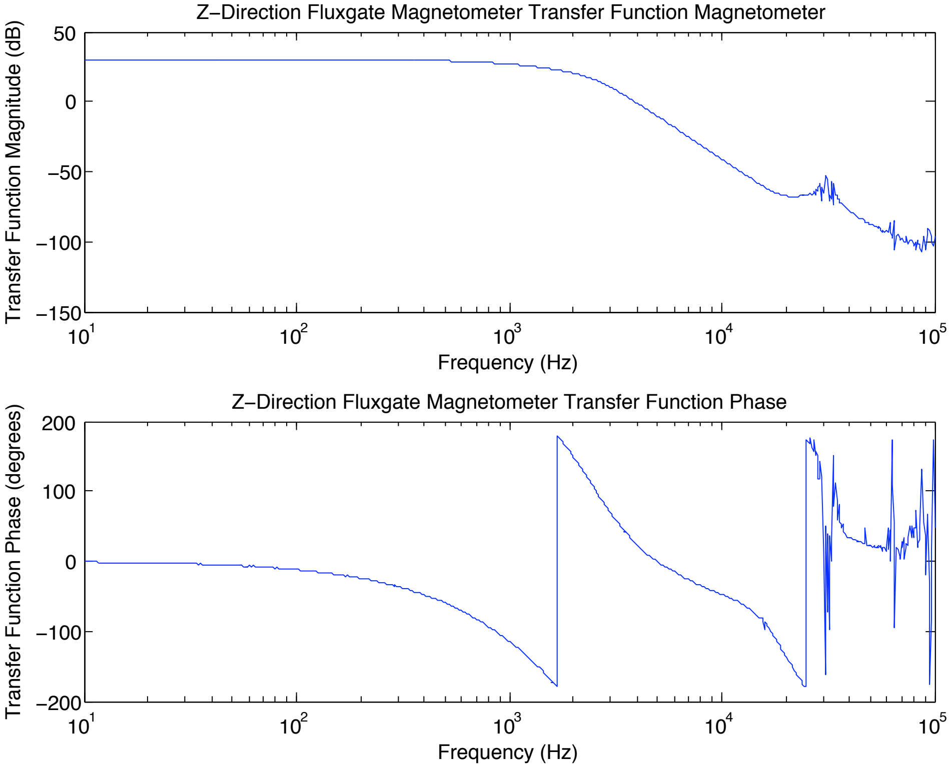
$$\begin{aligned} |\frac{V_o}{V_i}| &= \mu_o \frac{\alpha N}{h} \frac{cos(\omega t)}{R^2 + \omega^2 L^2} \\ &\approx \mu_o \frac{\alpha N}{hR} \text{ for } \omega \ll \frac{R}{L} \end{aligned}$$

We may determine that ω is sufficiently small by examining the phase of the transfer function. The phase will be nearly zero for small ω.

A.4 Data

The transfer function for the z-axis magnetometer is shown below. During experimentation it became apparent that fringe affects were dominating the magnetic field measurements necessary in the x- and y-axis calibrations.

We see in the figure that while the phase becomes non-zero above approximately 10^2 Hz the magnitude of the transfer function is constant below approximately 10^3 Hz.



A.5 Experimental α value

We use matlab to average the transfer function magnitude for the frequency range 0 - 10^3 Hz.

$$C_z \quad = \quad 29.6637 \pm .6876 \text{ dB} = 30.4218 \pm 2.40618$$

From the theory section we are led to believe that

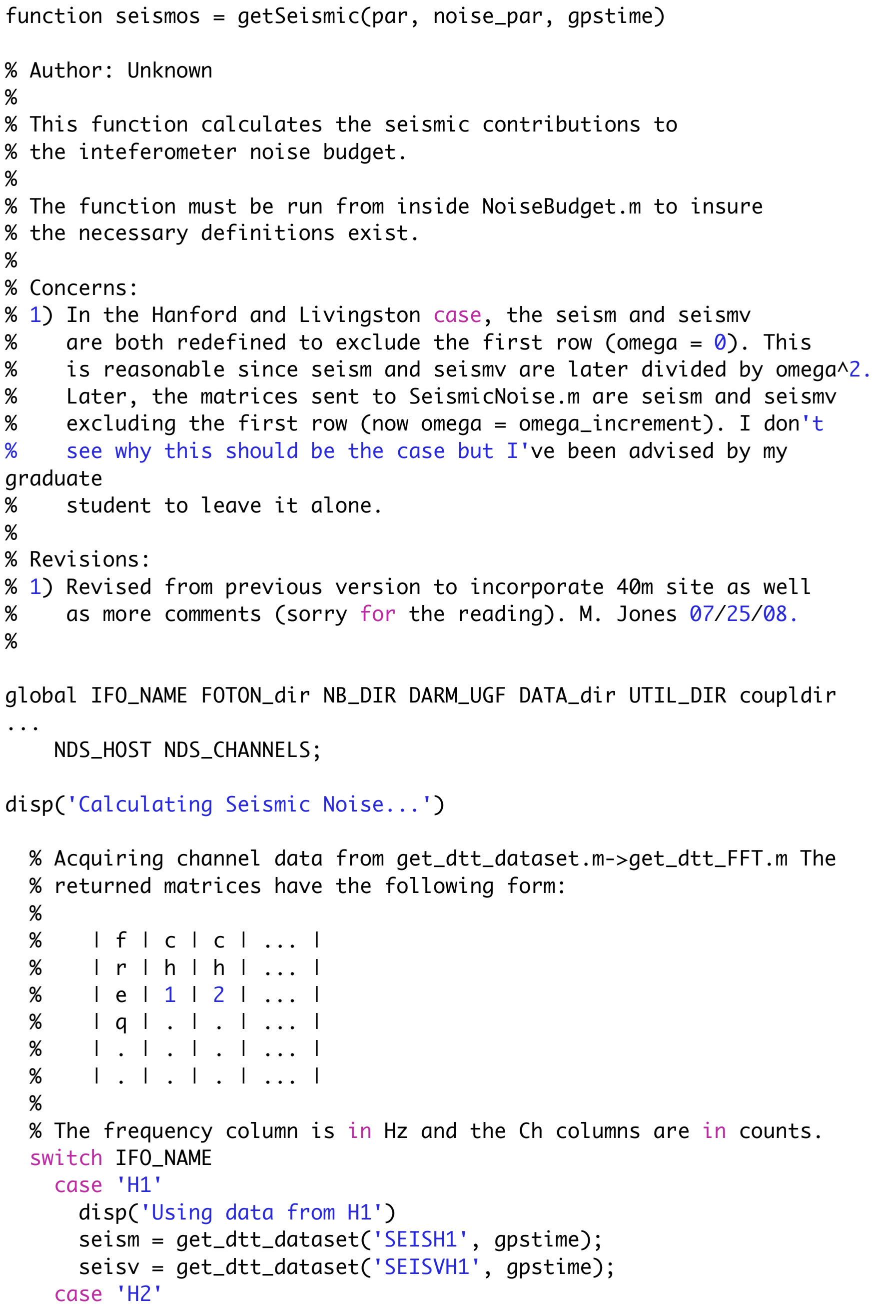
$$\alpha = C_z \frac{hR}{N\mu_0}$$

Therefore

$$\alpha = 14.1324 \frac{\text{nT}}{\text{mV}} \pm 1.12934 \frac{\text{nT}}{\text{mV}}$$

Due to the aforementioned difficulties in measuring the other axis of the magnetometer. We will make the assumption here that α is constant for all magnetometers.

B Revised Seismic Code



```
function seismos = getSeismic(par, noise_par, gpstime)

% Author: Unknown
%
% This function calculates the seismic contributions to
% the inteferometer noise budget.
%
% The function must be run from inside NoiseBudget.m to insure
% the necessary definitions exist.
%
% Concerns:
% 1) In the Hanford and Livingston case, the seism and seismv
%    are both redefined to exclude the first row (omega = 0). This
%    is reasonable since seism and seismv are later divided by omega^2.
%    Later, the matrices sent to SeismicNoise.m are seism and seismv
%    excluding the first row (now omega = omega_increment). I don't
%    see why this should be the case but I've been advised by my
graduate
%    student to leave it alone.
%
% Revisions:
% 1) Revised from previous version to incorporate 40m site as well
%    as more comments (sorry for the reading). M. Jones 07/25/08.
%

global IFO_NAME FOTON_dir NB_DIR DARM_UGF DATA_dir UTIL_DIR coupldir
...
    NDS_HOST NDS_CHANNELS;

disp('Calculating Seismic Noise...')

  % Acquiring channel data from get_dtt_dataset.m->get_dtt_FFT.m The
  % returned matrices have the following form:
  %
  %    | f | c | c | ... |
  %    | r | h | h | ... |
  %    | e | 1 | 2 | ... |
  %    | q | . | . | ... |
  %    | . | . | . | ... |
  %    | . | . | . | ... |
  %
  % The frequency column is in Hz and the Ch columns are in counts.
  switch IFO_NAME
    case 'H1'
      disp('Using data from H1')
      seism = get_dtt_dataset('SEISH1', gpstime);
      seisv = get_dtt_dataset('SEISVH1', gpstime);
    case 'H2'
```

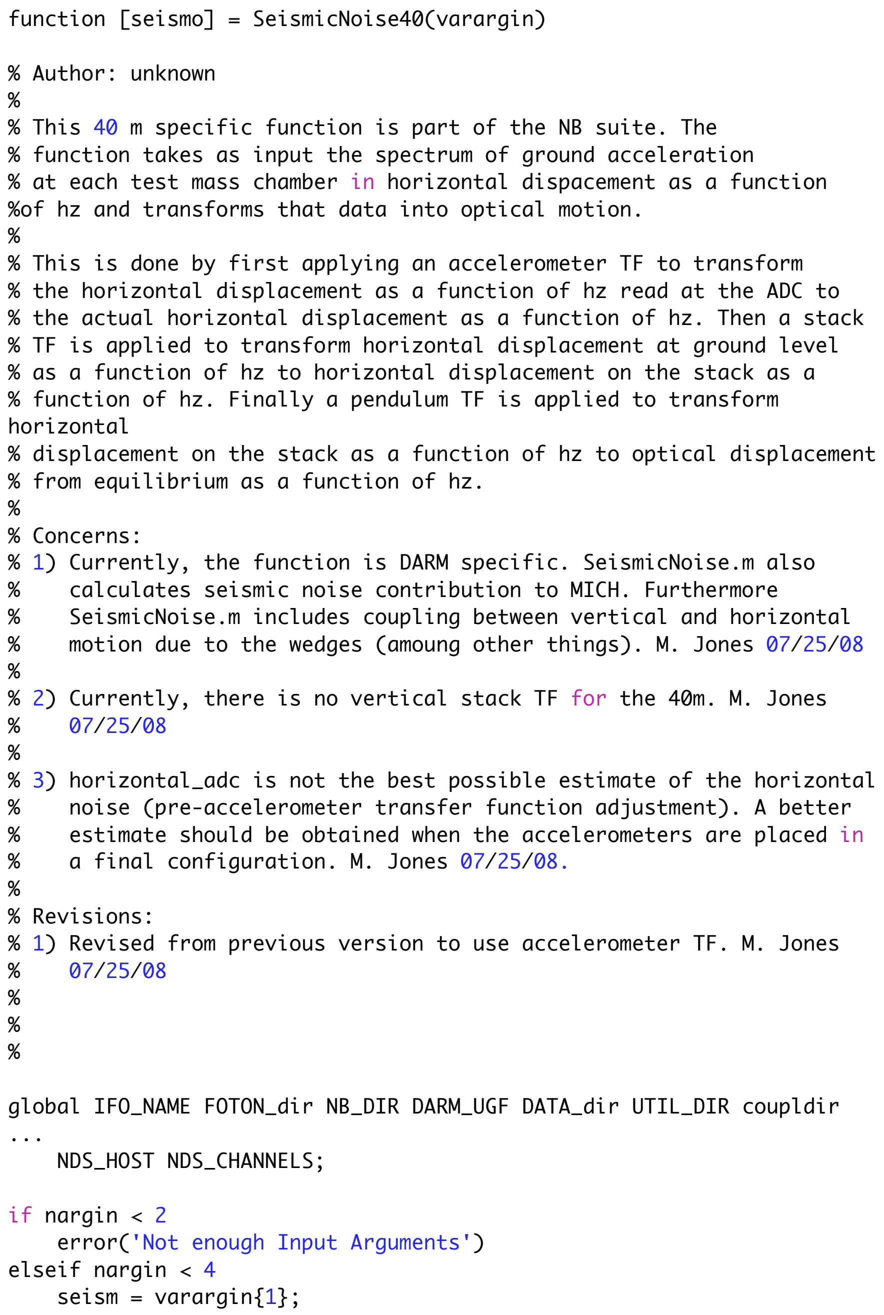
Figure 4: Revised getSeismic.m Function Page 1

```
   disp('Using data from H2')
   seism = get_dtt_dataset('SEISH2', gpstime);
   seisv = get_dtt_dataset('SEISVH2', gpstime);
  case 'L1'
   disp('Using data from L1')
   seism = get_dtt_dataset('SEISL1', gpstime);
   seisv = get_dtt_dataset('SEISVL1', gpstime);
  case 'C1'
   disp('Using data from C1')
   seism = get_dtt_dataset('SEISC1', gpstime);
   seisv = get_dtt_dataset('SEISVC1', gpstime);
  otherwise
   disp('unknown IFO')
 end

 % It is necessary to convert the data returned from
get_data_dataset.m which is
 % in cts as a function of hz to horizontal displacement as a function
of hz.
 % The need is fulfilled by the constant seism_dc which transforms
counts to
 % horizontal acceleration. Then the ground is assumed to be a simple
harmonic
 % oscillator such that (displacement) = (acceleration / (omega^2)).


 if (~strcmp(IFO_NAME, 'C1'))
  seism_dc = 4e-7; % calibration of accelerometers in m*cts/s^2.

  seism = seism(2:end,:);
  seisv = seisv(2:end,:);

  %Horizontal Direction
  tmp = seism_dc ./ seism(:,1).^2;
  tt1 = tmp(cumsum(ones(length(tmp),5),1));
  tt1(:,1) = 1;
  seism = seism .* tt1;

  % Vertical Direction
  tmp = seism_dc ./ seisv(:,1).^2;
  tt2 = tmp(cumsum(ones(length(tmp),5),1));
  tt2(:,1) = 1;
  seisv = seisv .* tt2;

  seismos = SeismicNoise(seism(2:end,:),seisv(2:end,:));
 elseif (strcmp(IFO_NAME, 'C1'))
  % The calibration constant seism_dc assumes the following:
  % 1) The Wilcoxon accelerometers are set to measure acceleration
```

Figure 5: Revised getSeismic.m Function Page 2

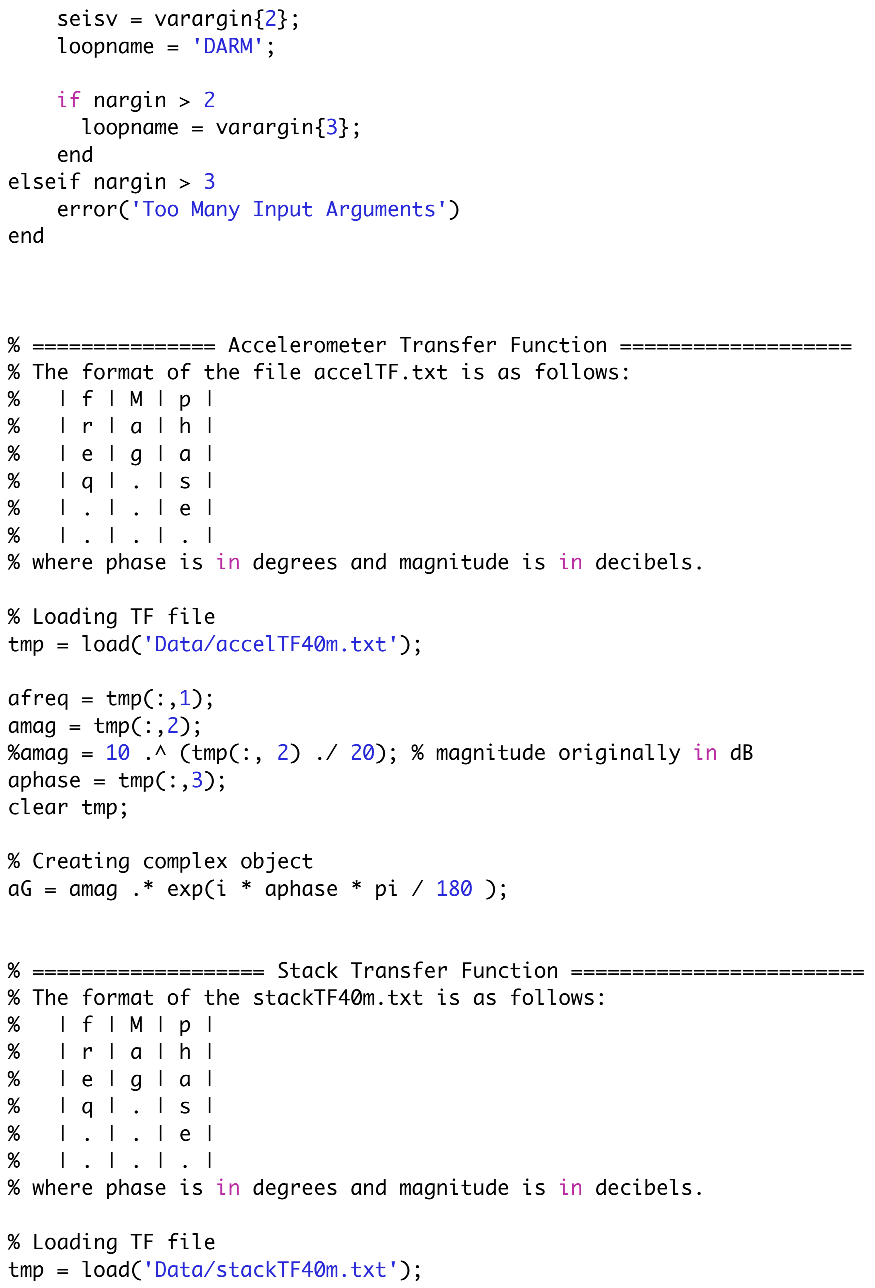
```
(and not
    %    velocity). The gain is 10 so that the conversion factor for
    %    accelerometer output is 100 V/g
    % 2) The ADC (analog to digital converter has range of (-2 V, 2 V)
    %    and 16 bit resolution yielding a conversion factor from the
    %    DAC input of 61.045 microV/count.
    % 3) The ground may be approximated accurately as a simple harmonic
    %    oscillator so that (displacement) = acceleration / (omega^2)

    seism_dc = 5.9824e-06; % calibration of accelerometers m*cts/s^2


    seism = seism(2:end,:);
    seisv = seisv(2:end,:);

    %Horizontal Direction
    tmp = seism_dc ./ seism(:,1).^2;
    tt1 = tmp(cumsum(ones(length(tmp),5),1));
    tt1(:,1) = 1;
    seism = seism .* tt1;


    % Vertical Direction
    tmp = seism_dc ./ seisv(:,1).^2;
    tt2 = tmp(cumsum(ones(length(tmp),3),1));
    tt2(:,1) = 1;
    seisv = seisv .* tt2;

    seismos = SeismicNoise40(seism(2:end,:), seisv(2:end,:));
  end
```

Figure 6: Revised getSeismic.m Function Page 3



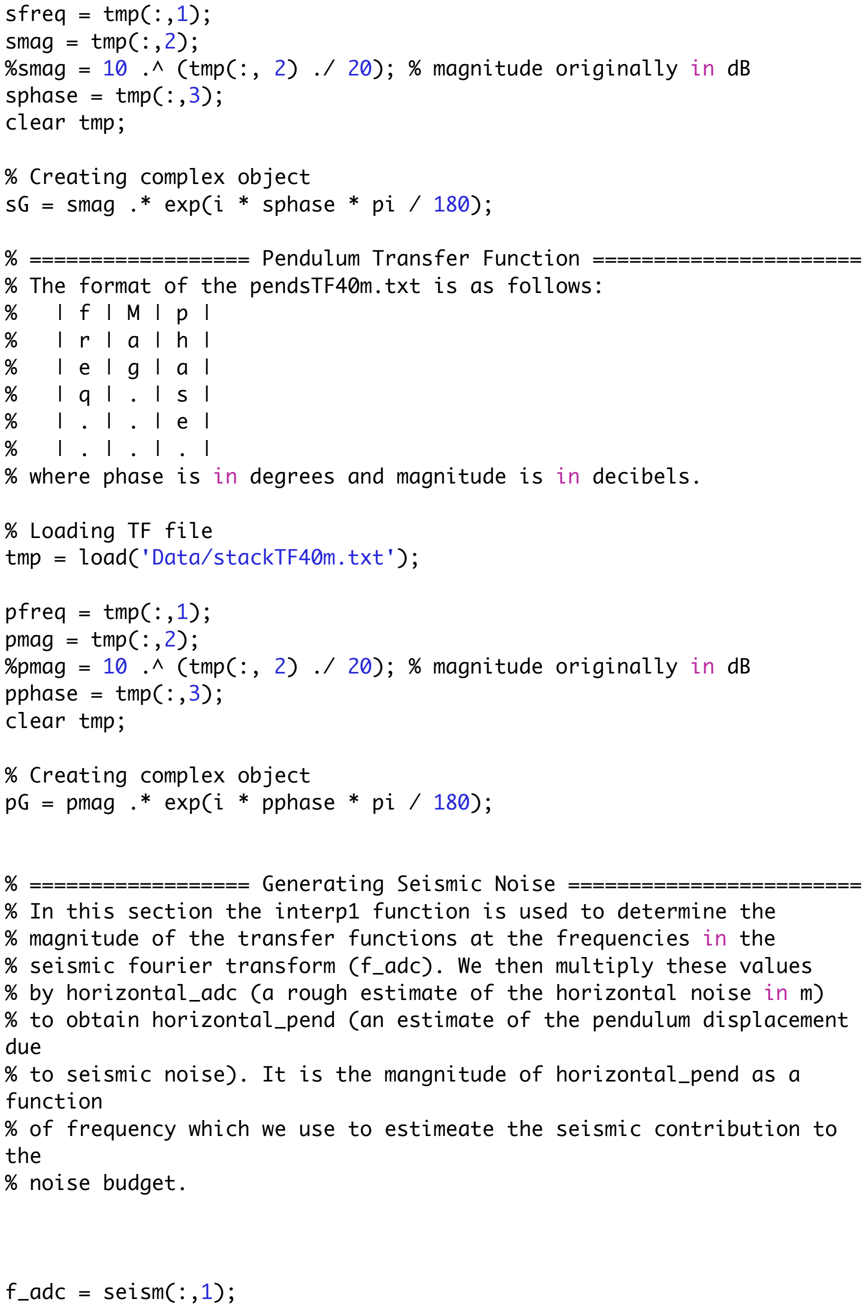
```
function [seismo] = SeismicNoise40(varargin)

% Author: unknown
%
% This 40 m specific function is part of the NB suite. The
% function takes as input the spectrum of ground acceleration
% at each test mass chamber in horizontal dispacement as a function
%of hz and transforms that data into optical motion.
%
% This is done by first applying an accelerometer TF to transform
% the horizontal displacement as a function of hz read at the ADC to
% the actual horizontal displacement as a function of hz. Then a stack
% TF is applied to transform horizontal displacement at ground level
% as a function of hz to horizontal displacement on the stack as a
% function of hz. Finally a pendulum TF is applied to transform
horizontal
% displacement on the stack as a function of hz to optical displacement
% from equilibrium as a function of hz.
%
% Concerns:
% 1) Currently, the function is DARM specific. SeismicNoise.m also
%    calculates seismic noise contribution to MICH. Furthermore
%    SeismicNoise.m includes coupling between vertical and horizontal
%    motion due to the wedges (amoung other things). M. Jones 07/25/08
%
% 2) Currently, there is no vertical stack TF for the 40m. M. Jones
%    07/25/08
%
% 3) horizontal_adc is not the best possible estimate of the horizontal
%    noise (pre-accelerometer transfer function adjustment). A better
%    estimate should be obtained when the accelerometers are placed in
%    a final configuration. M. Jones 07/25/08.
%
% Revisions:
% 1) Revised from previous version to use accelerometer TF. M. Jones
%    07/25/08
%
%
%

global IFO_NAME FOTON_dir NB_DIR DARM_UGF DATA_dir UTIL_DIR coupldir
...
    NDS_HOST NDS_CHANNELS;

if nargin < 2
    error('Not enough Input Arguments')
elseif nargin < 4
    seism = varargin{1};
```

Figure 7: Revised SeismicNoise40.m Function Page 1



```
    seisv = varargin{2};
    loopname = 'DARM';

    if nargin > 2
      loopname = varargin{3};
    end
elseif nargin > 3
    error('Too Many Input Arguments')
end



% =============== Accelerometer Transfer Function ===================
% The format of the file accelTF.txt is as follows:
%   | f | M | p |
%   | r | a | h |
%   | e | g | a |
%   | q | . | s |
%   | . | . | e |
%   | . | . | . |
% where phase is in degrees and magnitude is in decibels.

% Loading TF file
tmp = load('Data/accelTF40m.txt');

afreq = tmp(:,1);
amag = tmp(:,2);
%amag = 10 .^ (tmp(:, 2) ./ 20); % magnitude originally in dB
aphase = tmp(:,3);
clear tmp;

% Creating complex object
aG = amag .* exp(i * aphase * pi / 180 );


% =================== Stack Transfer Function ========================
% The format of the stackTF40m.txt is as follows:
%   | f | M | p |
%   | r | a | h |
%   | e | g | a |
%   | q | . | s |
%   | . | . | e |
%   | . | . | . |
% where phase is in degrees and magnitude is in decibels.

% Loading TF file
tmp = load('Data/stackTF40m.txt');
```

Figure 8: Revised SeismicNoise40.m Function Page 2



```
sfreq = tmp(:,1);
smag = tmp(:,2);
%smag = 10 .^ (tmp(:, 2) ./ 20); % magnitude originally in dB
sphase = tmp(:,3);
clear tmp;

% Creating complex object
sG = smag .* exp(i * sphase * pi / 180);

% ================== Pendulum Transfer Function ======================
% The format of the pendsTF40m.txt is as follows:
%   | f | M | p |
%   | r | a | h |
%   | e | g | a |
%   | q | . | s |
%   | . | . | e |
%   | . | . | . |
% where phase is in degrees and magnitude is in decibels.

% Loading TF file
tmp = load('Data/stackTF40m.txt');

pfreq = tmp(:,1);
pmag = tmp(:,2);
%pmag = 10 .^ (tmp(:, 2) ./ 20); % magnitude originally in dB
pphase = tmp(:,3);
clear tmp;

% Creating complex object
pG = pmag .* exp(i * pphase * pi / 180);


% ================== Generating Seismic Noise ========================
% In this section the interp1 function is used to determine the
% magnitude of the transfer functions at the frequencies in the
% seismic fourier transform (f_adc). We then multiply these values
% by horizontal_adc (a rough estimate of the horizontal noise in m)
% to obtain horizontal_pend (an estimate of the pendulum displacement
due
% to seismic noise). It is the mangnitude of horizontal_pend as a
function
% of frequency which we use to estimeate the seismic contribution to
the
% noise budget.



f_adc = seism(:,1);
```

Figure 9: Revised SeismicNoise40.m Function Page 3

```
horizontal_adc = sqrt(seism(:,2).^2 + seism(:,3).^2 + seism(:,4).^2 +
seism(:,5).^2);

TF1 = interp1(afreq, aG, f_adc, 'spline','extrap');
TF2 = interp1(sfreq, sG, f_adc, 'spline','extrap');
TF3 = interp1(pfreq, pG, f_adc, 'spline','extrap');

horizontal_gnd = abs(TF1) .* horizontal_adc;
horizontal_stk = abs(TF2) .* horizontal_gnd;
horizontal_pend = abs(TF3) .* horizontal_stk;

% ============================ Output ================================

switch upper(loopname)
    case 'DARM'
        horizontal_noise = sqrt(horizontal_pend .^ 2);
    otherwise
        error('Invalid Loop Name');
end

if nargout == 1
    ]
    seismo = [f_adc horizontal_noise];
else
end
```

Figure 10: Revised SeismicNoise40.m Function Page 4