Absolute length measurement for the cavities of the 40-m interferometer

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2 Task

- Measure absolute lengths of the cavities with precision of less than 1mm so that one can adjust the position of the suspensions as designed. This precision corresponds to dL/L of about 10^{-5}. It is also good to know these values for the modeling of the interferometer. If the procedure is applicable to LIGO and Adv LIGO, our trial will be more valuable.
- Target cavities:
  - Arm cavities: Inline arm and perpendicular arm
  - PRCs: Inline PRC and perp. PRC. (Individually and being averaged as recycled Michelson)
  - SECs: Inline SEC and perp. SEC. (Individually and being averaged as recycled Michelson)

3 Previous Trials

- **Trial at the 40m**

  There is a measurement result of the arm cavity length at the previous 40m setup in 1999 [3]. In the measurement, the unlocked cavity was swept and the FSR was measured from the PDH signal using the resonances of the phase modulation sidebands as a frequency reference. The obtained result was $dL = 4$mm against $L = 38.546$m. This corresponds to dL/L of 10^{-4}. This would have been limited by the linearity of the swinging and the readout of the zero crossings of the PDH method.

- **Trial at TAMA**

  There is also a measurement result of the arm cavity length at TAMA. The absolute length of the 300-m cavity was measured in 1998 [1]. In the experiment the phase modulation at the integer multiple of the FSR (~500kHz) was used, while the arm itself was locked using the PDH technique with the phase modulation at 15MHz. Although the method itself was equivalent to the scheme invented earlier by DeVoe et al [2], this was the first application to the long cavity. As the mode cleaner was not present this time, phase modulation at any frequency could be applied. The measured cavity length was $L = 299943980\mu$m with the precision of $dL = 1\mu$m. This corresponds to dL/L of 3×10^{-9}. Since this technique is a phase measurement of the modulation sideband beam, this strain precision is very good. Actually it is too good for our goal.

4 Method

We use a simplified scheme of [1] and [2]. Secondary laser beam is set to be resonant to the cavity, while the main laser is locked to the cavity at the same time. Frequency difference obtained from the beating between the two resonant lasers shows the integer multiple of the cavity FSR.

The method to modulate the PSL, instead of using the secondary laser, is very difficult owing to the presence of the mode cleaner, which reject modulation sidebands except for its resonances.

First trial will be performed for the 40-m arm cavity as it is the simplest among the cavities. Then we will move to the other cavities.

5 Key points of the experiment

5.1 Beam injection

5.1.1 Place to inject

There are several possiblity for the injection points of the secondary beam: the AR coating of the ITMs (POX or POY), the ETMs, the dark port, and so on.

The dark port is expected to be the best place to put in spite of the presence of an attenuation by the misaligned SRM (T=7%). The ETMs provides direct access to the arm cavities, however, the arm cavity is strongly under-coupled. This will make determination of resonances difficult, particulary when perfect alignment is not yet obtained. Injection from POX or POY suffers from the big attenuation of the AR coatings.

A simple calculation shows the expected beating level when the dark port is used for the injection. Assume that both of the recycling mirrors are misaligned. The main beam incident on the recycling mirror is 1W, and the injected beam is assumed to have the output of 100mW. The optical parameters of the 40m was took from the references [4]. Note that the loss of 60ppm was used instead of 37.5ppm per reflection as the actual power recycling gain obtained was 60% of the design. The arm cavity is supposed to have the power gain of 756 and the power transmittance of 0.011.

From these values, the power distributions of the main beam and the injected beam are obtained as displayed in Figure. 1 and Figure. 2. Apparently, the power of the injected beam is 0.1 times of the main beam power because of the symmetry of the IFO. When the power ratio of the two beating electric field is R, the beating level is given by $4\sqrt{R}/(1+R+2\sqrt{R})$ [1]. The expected maximum beating level is

$$R = 40\mu\mathrm{W}/400\mu\mathrm{W} = 0.1 \Rightarrow 73\%$$

This value looks sufficient for our purpose. We will check an actual injected power and its mode matching.

Another point to note is that the beat signal can also be obtained from the bright port or the reflection to the AS port. Since we don't need to construct the beating optics separately, these signals will be useful for the PLL (phase locked loop) which stabilizes the frequency difference of the two lasers.

5.1.2 Mode matching of the secondary laser?

We must make the laser beam with a long Rayleigh range using a short telescope on the optical table? How can it be difficult?

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[1] $E_{\mathrm{total}} = E_1 + E_2 = 1 + \sqrt{R}e^{I\phi}$, $P = EE^* = 1 + 2\sqrt{R}\cos\phi + R$, $P_{\min} = 1 - 2\sqrt{R} + R$, $P_{\max} = 1 + 2\sqrt{R} + R$, $1 - P_{\min}/P_{\max} = 4\sqrt{R}/(1 + 2\sqrt{R} + R)$

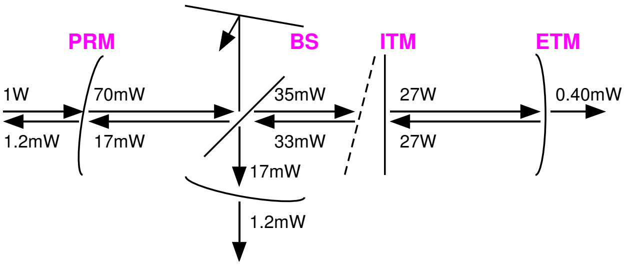


Figure 1: Power distribution of the main beam

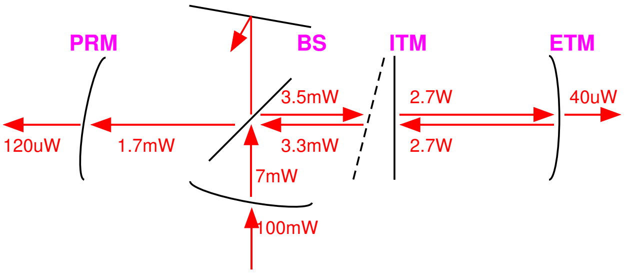


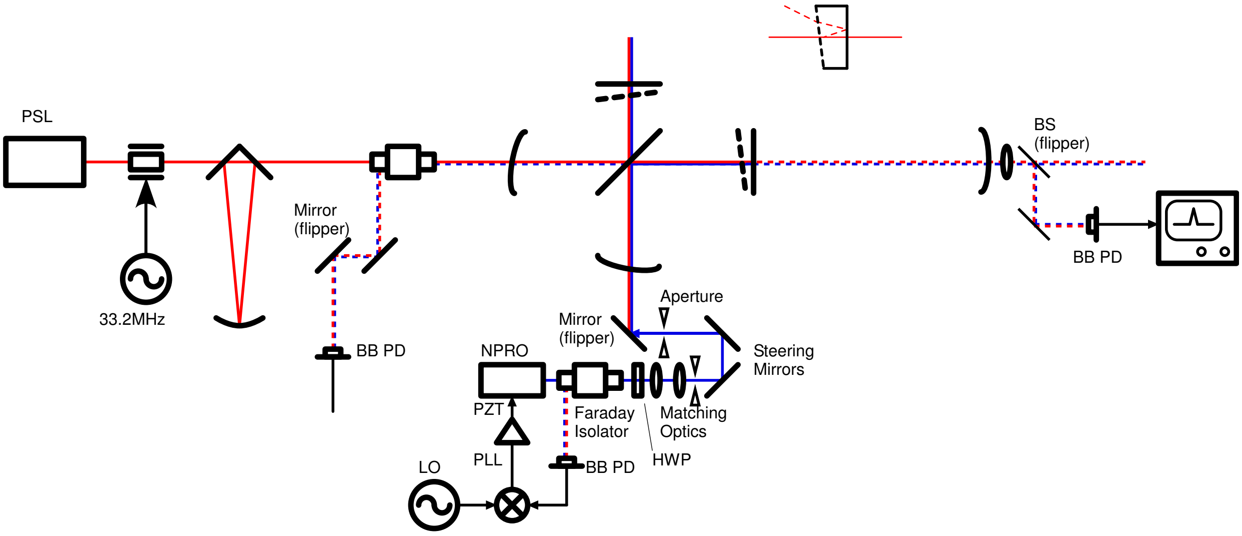
Figure 2: Power distributions of the injected beam from the dark port

5.1.3 How to take an alignment of the secondary beam in terms of the cavity

In principle we can adjust the alignment of the incident beam with two steering mirrors. But how do we make the initial alignment when there is no resonance? Use the main beam and the irises? Do we use any interference of the first beam and the secondary beam?

5.2

- **How to detect the resonance of the secondary beam?**
  - At the transmittion? At the reflection? What is the difference?
  - We need a broad band detector that can be sensitive 3-10MHz.
  - We take an intensity of the beating which will be maximum at the resonance of the secondary beam. Does it have enough precision for $dL/L \sim 10^{-5}$?
- **Use of PLL**
  - The relative laser frequency of the two lasers are fluctuating. A typical line width of free running NPROs is about 10kHz. This means that we need a phase locked loop between two lasers.
  - The secondary laser will be located physically separated place from the PSL. How do we get the beating of them? Fiber?
- **The case of low finesse cavity (PRC/SEC)**
- **Effect of the beamsplitter (for PRC/SEC)**



Use pinhole/SRM/ITM/PRM for the injection alignment

Figure 3: XXX

6 Key points of the experiment

7 Side product

- We acquire a technique to inject aux laser beam into the cavities. It wil be useful for the future possible injection of the aux green laser for lock acquisition.

8 Reproduction of the cavity sweep experiment

Measure of the Macroscopic Cavity Lengths of the Caltech 40m Interferometer
by Alberto Stochino, and Koji Arai

We measured the lengths of both the X and Y arm cavities of the LIGO 40m Interferometer at Caltech by means of the RF modulation with a precision of 0.2.

8.1 The technique

Suppose we have a laser carrier of wavelength λ_c resonating in a Fabry-Perot cavity of length L. Suppose now we slowly change the laser frequency and at the same time we move the end mirror to keep the cavity on resonance so that the number of half wavelengths contained in the entire length does not change. If λ_s is the final wavelength of the laser, the mirror has to be displaced by ΔL:

$$\Delta L = \left[\left(\frac{L}{\lambda_c/2} \right) \cdot \frac{\lambda_s}{2} - L \right].$$

If, instead of variating the frequency, we suppose to have two laser components of wavelength λ_c and λ_s coexisting in the cavity, then ΔL is the displacement of the mirror necessary to makeλ_s resonant with the same number of half wavelengths. The case is analogous to that in which λ_c and λ_s are the carrier and the sideband wavelength of a modulated laser.

Starting when the carrier is resonant, we want to extend the cavity length displacing the end mirror by ΔL until the sideband becomes resonant. If, while sweeping the mirror position, we look at the transmitted power through the mirror or at the demodulated reflected power, we observe peaks when either one of the component is resonant. Naming ΔL_{fsr} the distance between two adjacent peaks of the carrier, this is equal to half wavelength of the carrier and we have:

$$\frac{\Delta L}{\Delta L_{fsr}} = \frac{\left(\frac{L}{\lambda_c/2}\right) \cdot \frac{\lambda_s}{2} - L.}{\lambda_c/2} = \frac{\nu_m}{\nu_{fsr}} \cdot \frac{\nu_c}{\nu_s} \approx \frac{\nu_m}{\nu_{fsr}}$$

where ν_m is the modulation frequency and the last approximation holds for the case of a RF modulation. From the measurement of $\Delta L/\Delta L_{fsr}$, knowing the carrier and the sideband wavelength, we can calculate the free spectral range ν_{fsr} and then obtain the cavity length L as:

$$L = \frac{c}{2\nu_m} \frac{\Delta L}{\Delta L_{fsr}}.$$

Then the error on L is:

$$\delta L = \frac{c}{2\nu_m} \cdot \left(\frac{\delta\left(\Delta L\right)}{\Delta L_{fsr}} + \frac{\Delta L \cdot \delta\left(\Delta L_{fsr}\right)}{\Delta L_{fsr}^2} \right)$$

where δindicates the error on the single quantities.

8.2 The measurement

We cannot have direct measures of ΔL or ΔL_{fsr} with the needed resolution. However we can evaluate their ratio from that of the time intervals between the occurrences of the correspondent resonances on a time series plot of either the transmitted or the reflected power. The ratio of displacements and the ratio of time intervals turn out to be the same if the mirror moves at a constant velocity. If the movement of the mirror is a pendulum-like oscillation, we can assume the velocity as constant in a certain interval around the equilibrium point, where the acceleration is zero.

Two sidebands are introduced by the modulation of the laser. Figure 4 shows the series of a carrier peak followed by two sideband peaks. Since we approximately know that $L \approx 38.5m$ we expect to count 8 carrier modes in between the carrier and the sideband corresponding to the same longitudinal mode of the cavity. Looking at the time series plot, we always have two sideband resonances between two carriers. One is the lower sideband of a carrier mode located 8 carrier resonance peaks before in the time series, and the other is the upper sideband of a carrier mode located 8 carrier resonances after in the plot. Before measuring ΔL we have to distinguish the upper sideband from the lower one. The only way to decide is a posteriori, that is estimating the cavity length from both the resonances and choosing the one that gives the most reasonable value for the cavity length considering the value known for it.

We chose to read the positions of the peaks in the plot from the point in which the demodulated output crosses zero. A different measurement of ΔL_{fsr} was obtained for each sideband resonance averaging over the 8 intervals between the carrier resonances. That enabled us to better keep into account for the change of velocity of the mirror during time.

From the average on 12 measures, and using the values $\lambda c = 1064nm$, $\nu_m = 33.195439Mhz$ and $\delta\left(\Delta L\right) = \delta\left(\Delta L_{fsr}\right) = 5 \times 10^{-5}s$ for the parameters, we obtained the following numbers for the cavity lengths $L_x = (38.30/38.45 \pm 0.08)m$, $L_y = (38.16/38.70 \pm 0.08)m$ in which the the two possibilities are obtained depending on which of the sideband resonance belongs to the lower sideband.

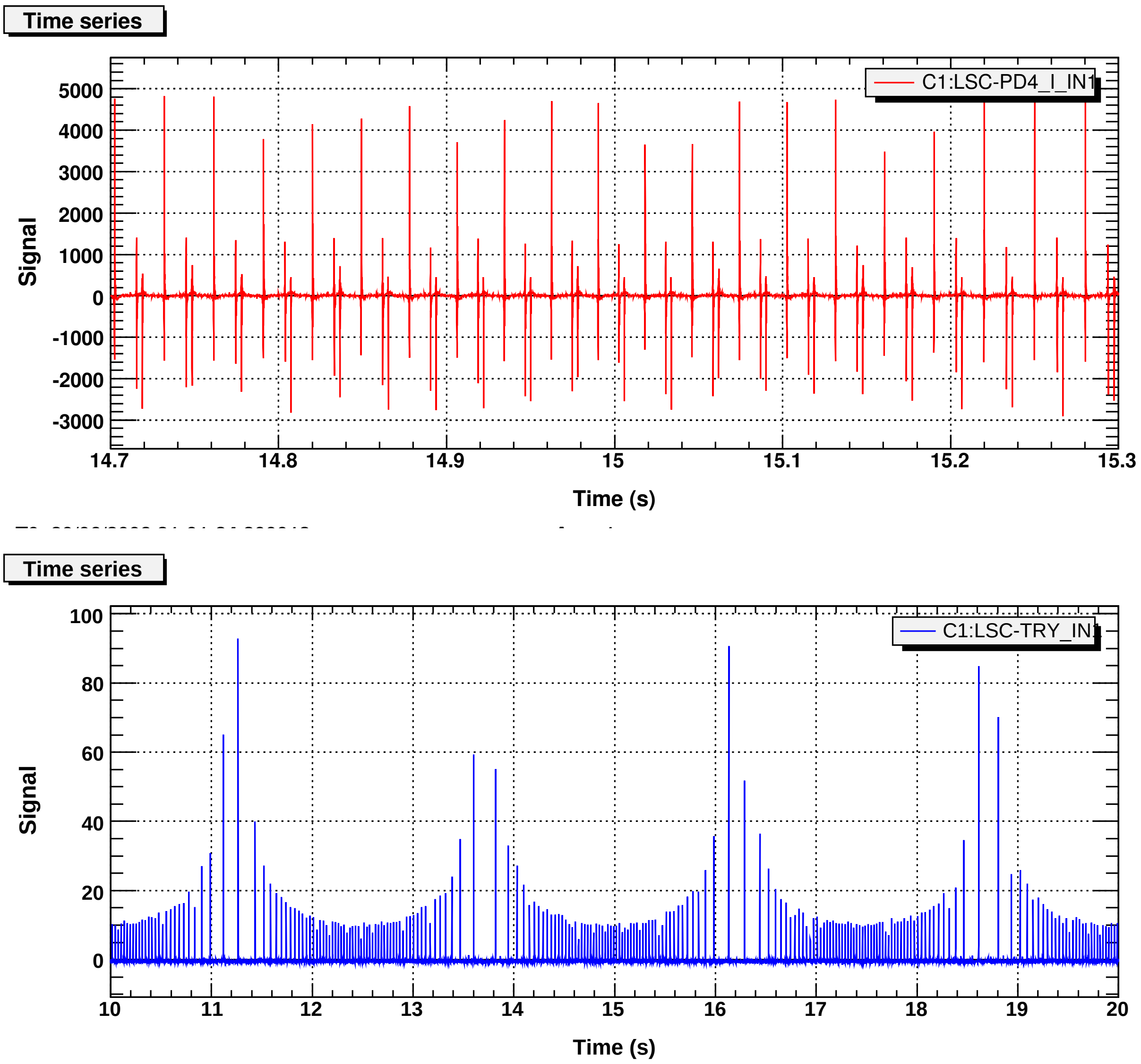


Figure 4: Time series plot of the demodulated reflected output (upper figure) and of the transmitted output (lower figure). The change in amplitude well shown in the second plot is due to the different velocity of the mirror during time compared to a build-up time of the cavity of about 1 ms. We measure more intensity for the resonances in which the mirror stays for longer.

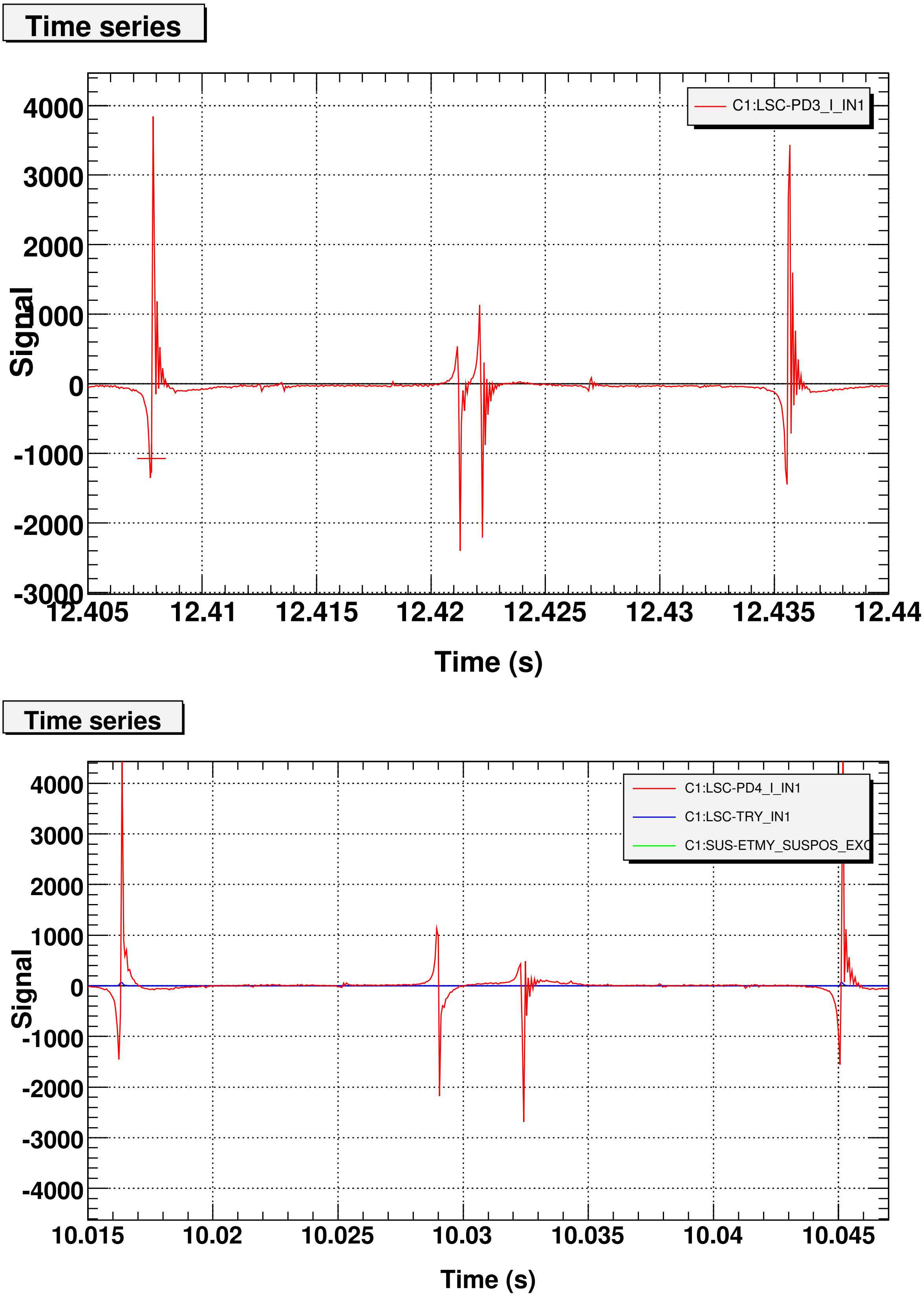


Figure 5: The plots in the figure show the zoom on a time interval of the demodulated output power of the X (upper plot) and the Y (lower plot) arm cavities. It is no possible a priori to distinguish which of the sideband resonances is the upper or the lower one. The separation between the sideband resonances is due to the different lenth of the to arm cavities,

8.3 Non-optical measurement

As described in the previous sections, the results of the cavity length measurement using cavity sweeping has inevitable ambiguity. There are two solutions for Each cavity length depending on which resonance do we take as an upper sideband.

In order to exclude this ambiguity a primitive non-optical measurement using a tape and photos was performed: The lengths between the ITM/ETM chambers were measured by the scaled tape. Also, the actual positions of the suspensions were estimated from the photographs through the optical windows.

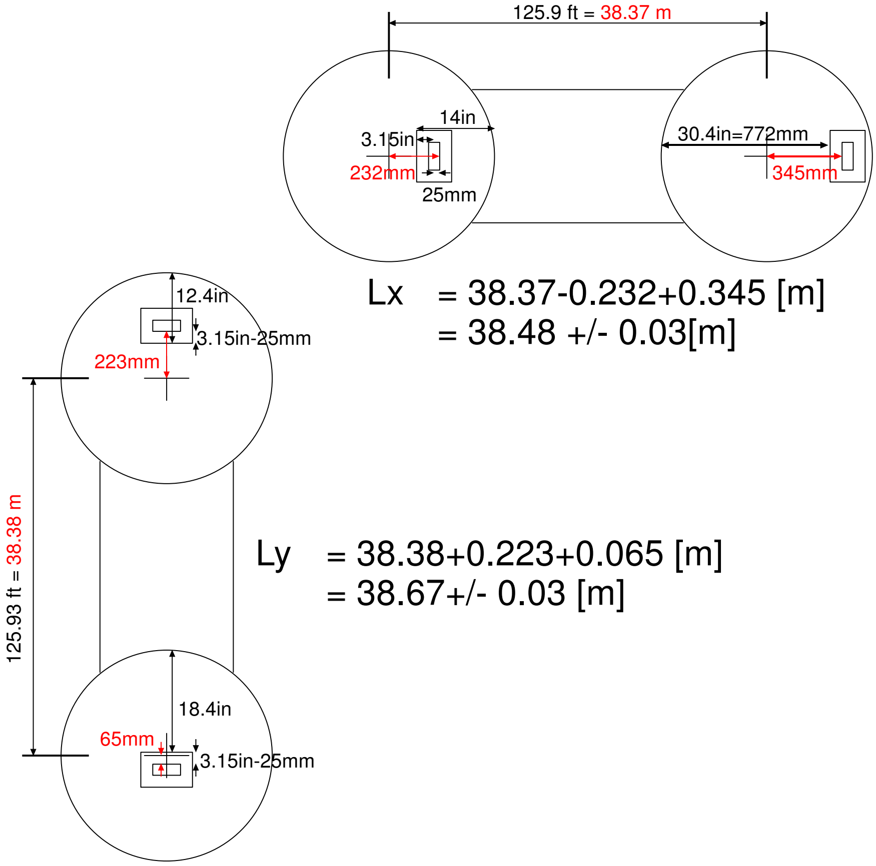


Figure 6: The actual mirror surface position estimated from the tape measurement and the photographs.

The results are shown in Figure 6 and Figure 6. The lengths of the X/Y arm are known to be 38.48+-0.03 / 38.67+-/0.03 [m].

Combined with the cavity sweeping results, we could exclude the shorter lengths of the values of the cavity sweeping experiment. i.e. The Y arm is longer than the X arm about 0.2 m.

These approximate lengths will be used in the further precise measurements which use precise scans of the FSR frequencies.

References

[1] A. Araya et al, Appl. Opt. **38** (1999) 2848-2856.
Absolute-length determination of a long-baseline fabry-perot cavity by means of resonating modulation sidebands.

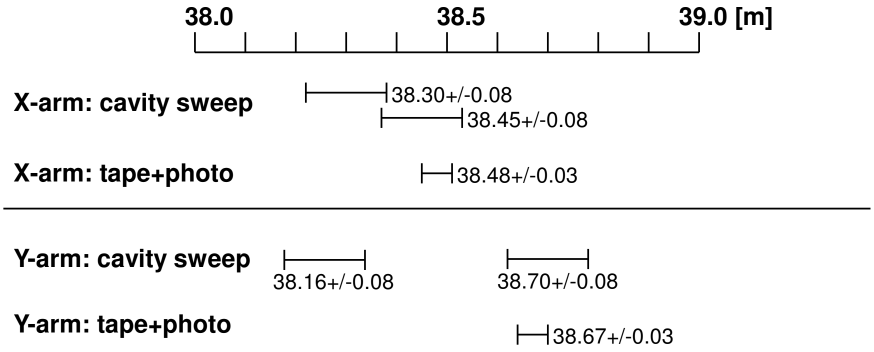


Figure 7: The arm length determination with the cavity swinging and the tape measurement.

[2] R. G. DeVoe and R. G. Brewer, Phys. Rev. A **30** (1984) 2827-2829.
Laser frequency division and stabilization.

[3] M. Rakhmanov, M. Evans, and H. Yamamoto, Meas. Sci. Technol. **10** (1999) 190-194.
An optical vernier technique for in situ measurement of the length of long Fabry-Perot cavities; Matt Evans, Malik Rakhmanov, and Hiro Yamamoto, LIGO Document T970174-00-D
`http://www.ligo.caltech.edu/docs/T/T970174-00.pdf`
Interferometric Vernier Technique for Measuring the Lengths of LIGO Fabry-Perot Resonators

[4] `http://www.ligo.caltech.edu/c̃it40m/Docs/40m_params.pdf`
Summary of 40m IFO parameters; Robert Ward, private communication.