Phase-lock control considerations for coherently combined lasers

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Fundamental performance limitations of a phase-lock control loop used to coherently combine the output of two lasers are presented. The phase-lock loop is designed to lock the differential phase (frequency and phase) between the two lasers to a specified reference phase. An optical heterodyne configuration is used to determine the differential phase of the laser pair, which in turn is compared with the reference phase to create an error voltage. The error voltage is filtered and used to frequency modulate one of the lasers in an attempt to null the error. An integro-differential loop equation, valid for the linear operating range, is derived in terms of the reference phase, the heterodyne measurement noise, and the various laser phase instabilities. The solution of the equation results in an expression for the phase error variance in terms of the closed-loop noise equivalent bandwidth W_H. An expression for the value of W_H which minimizes the phase error variance is developed. In addition to the noise effects, the steady-state and dynamic performance of the loop is examined for different loop filters and modulation formats. A design example pairing a CO_2 waveguide and conventional laser is presented. Implications for coherent laser arrays are discussed.

I. Introduction

A well-known method of generating periodic, optical pulse trains with high peak power, and extremely narrow pulse widths is the mode-locked laser.[1] However, the modulation characteristics are largely dependent on the laser's cavity parameters. For example, the pulse repetition rate of a mode-locked laser is determined primarily by the cavity length. Thus, the modulation format is inflexible.

Conceptually, it is easy to think of each laser mode as being generated in individual laser cavities. So, a generalized concept of mode-locking could be the coherent combination of the outputs of an array of single-mode, but electronically phase-controlled, lasers. Such a configuration has a potential modulation format that is much more flexible. For the special case of each individual laser offset from the next by a constant frequency, such a configuration will result in the familiar mode-locked waveforms, as was recently experimentally demonstrated.[2] A discussion of the possible types of waveforms and their potential applications is presented elsewhere.[3]

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The key requirement for such a system is the precise control of the relative frequencies and phases of the individual lasers of the array. A logical first step in the construction of such an array is the phase locking of a single pair of lasers. This paper presents fundamental performance limitations and design considerations for a phase-lock feedback control loop designed to lock the frequency/phase difference between the output beams of two single-mode lasers to a desired controllable value. An integro-differential equation, valid for the linearized operating region of the loop, is derived in terms of the desired frequency/phase control, the laser phase instabilities, and other loop component noises and instabilities. Expressions for the power spectra of the various noises and instabilities are used to solve the integro-differential equation for the phase error variance in terms of the closed-loop noise equivalent bandwidth of the system. The phase error variance expression can be used to determine the optimum (i.e., minimum variance) closed-loop bandwidth. In addition to noise effects, system performance parameters (such as loop acquisition time, frequency pull-in range, and steady-state error) are related to the loop filter characteristics and the closed-loop bandwidth for various desired control schemes (such as step or ramp changes in frequency). A detailed design example for the two-laser control loop is presented to illustrate the effects of the various noise and control parameters on the over-all phase-lock performance.

II. Two-Laser Phase-Lock Problem

When one mentions the problem of phase-lock of two sinusoidal sources, the first configuration that comes to mind is the well known phase-lock loop (PLL), shown in Fig. 1. Ideally, two lasers could be placed in phase-lock by letting the output field of one laser be the PLL input signal and letting the other, its frequency being voltage adjustable, act as the voltage-controlled oscillator (VCO). Unfortunately, this scheme is not feasible, since there are no known methods for direct measurement of laser field $\mathcal{E}(\bar{r},t)$. All schemes detect the intensity or power density of the field $|\mathcal{E}(\bar{r},t)|^2$, thus losing all phase information. Furthermore, the extremely high laser frequencies, on the order of 10^{14} Hz, would also present difficult electronic problems.

To circumvent these problems, a common procedure is to sum (or interfere) the output fields of the two lasers onto a single detector. The output of the detector will include a signal with a phase equal to the difference of the laser phases plus low-frequency terms. The phase difference, or beat frequency, is more readily handled by available electronics. Typically, one laser has a stable phase so that the phase instabilities of the other can be examined.[4]

Another reason this procedure is so attractive is that excellent noise performance can be obtained with simple postdetector processing. (This well-known method, known as heterodyne detection, will be discussed later.) We are now ready to discuss the two-laser phase control problem (in relation to the well-known PLL) in detail.

A. Two-Laser PLL: Preliminary Discussion

The basic PLL of Fig. 1 can now be adapted to lock the phase difference between two lasers to a reference phase, as shown in Fig. 2. The entire laser pair and detector combination acts as the VCO. The filtered error voltage modulates the frequency of one laser only, but this is translated to a modulation in the differential phase seen at the detector. The important aspect of the two-laser loop is that the differential phase between the two lasers is locked to a reference phase; the absolute phase of the laser pair remains uncontrolled except by the nature of the lasers themselves and thus is free to fluctuate.

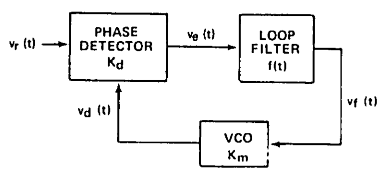

Fig. 1. Basic phase-lock loop.

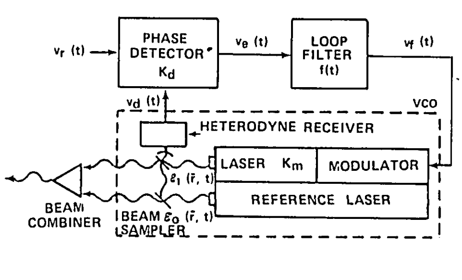

Fig. 2. Two-laser differential phase control loop.

The loop input reference signal has the form

$$v_r(t) = V_r \sin\Phi_r(t), \tag{1}$$

where

$$\Phi_r(t) = (2\pi f_d t + \phi_d) + \theta_r(t). \tag{2}$$

The first term, $(2\pi f_d t + \phi_d)$, is the constant portion of the reference phase and represents the VCO quiescent frequency. (All derivations will be made with respect to the VCO quiescent frequency f_d.) The second term, $\theta_r(t)$, is the modulated term and consists of two portions:

$$\theta_r(t) = \zeta(t) + \gamma_r(t), \tag{3}$$

where $\zeta(t)$ is the control modulation, and $\gamma_r(t)$ is a zero mean random process representing the reference phase instabilities.

The voltage-controlled oscillator (VCO) consists of the two lasers, one of which is modulated by the filtered error voltage, and a heterodyne receiver (HR) that utilizes samples of their output fields. The output fields of the lasers are combined for use in the far field, as seen in Fig. 2. However, samples of the fields are taken for use in the control loop. These sampled fields are assumed to be planar with constant amplitudes. Thus the sampled fields for the reference and modulated lasers are, respectively,

$$\mathcal{E}_0(\bar{r},t) = A_0 \cos\Phi_0(t), \tag{4}$$

$$\mathcal{E}_1(\bar{r},t) = A_1 \cos\Phi_1(t). \tag{5}$$

The phase of the reference laser is

$$\Phi_0(t) = (2\pi f_0 t + \phi_0) + \gamma_0(t), \tag{6}$$

where $(2\pi f_0 t + \phi_0)$ is the operating phase, and $\gamma_0(t)$ is a zero-mean, Gaussian random process describing the measurable reference laser instabilities. Its statistical description is discussed in detail in a later section. Since this laser is not modulated, f_0 is always the quiescent frequency f_{0q}, that is,

$$f_0 \equiv f_{0q}. \tag{7}$$

The phase of the modulated laser is

$$\Phi_1(t) = (2\pi f_1 t + \phi_1) + \gamma_1(t), \tag{8}$$

where $(2\pi f_1 t + \phi_1)$ is the operating phase, and $\gamma_1(t)$ is again a zero-mean, Gaussian random process representing that laser's instabilities. More specifically, the operating frequency f_1 consists of a constant quiescent frequency f_{1q} portion and a portion modulated by the filtered error. The modulated laser is frequency modulated by applying the filtered error voltage $v_f(t)$ to an internal electrooptic or external piezoelectric crystal, which changes the effective cavity length. The instantaneous frequency of the modulated laser is

$$\dot{\Phi}_1(t) = 2\pi[f_{1q} + K_m v_f(t)] + \dot{\gamma}_1(t), \quad (9)$$

where K_m is the modulation gain due to the crystal in hertz per volt.

The two sampled laser fields are directed into a heterodyne receiver (HR). The output of an HR is well known and will simply be stated in this paper. The results are conditioned on the following assumptions:

(1) The fields impinging on the detector are collinear, planar, of constant magnitude, and normally incident.

(2) The combined beam is incident on a detector of active area A_d.

(3) The fields, traveling in free space, have units of $V/m\sqrt{\Omega}$, and without loss of generality,

$$A_0 \gg A_1. \quad (10)$$

(4) Finally, the bandpass filter (BPF) is ideal with impulse response $b(t)$ and transfer function $B(j2\pi f)$. The filter $|B(j2\pi f)|^2$ has magnitude $1/C_{01}^2$, bandwidth B_{01} Hz, and center frequency

$$f_d = f_{1q} - f_{0q}. \quad (11)$$

The filter characteristic is shown in Fig. 3.

With these assumptions the output of the HR is[5]

$$v_d(t) = V_d \cos\Phi_d(t) + n(t), \quad (12)$$

where

$$V_d = \frac{q\eta A_d A_0 A_1 R}{h f_0 C_{01}}, \quad (13)$$

$$\Phi_d(t) = \Phi_1(t) - \Phi_0(t). \quad (14)$$

In the above, q is the charge of an electron, η is the quantum efficiency of the detector, h is Planck's constant, R is the effective load resistance of the detector, and $n(t)$ is a zero-mean, Gaussian random process. The HR noise arises from the fundamental quantum nature of the optical detection process and should not be confused with the noise arising from the laser phase instabilities. The statistical nature of $n(t)$ will be examined in a later section.

The phase of the HR output can be rewritten as

$$\Phi_d(t) = \Phi_1(t) - \Phi_0(t) = 2\pi f_d t + \phi_d + \theta_d(t), \quad (15)$$

where

$$f_d = f_{1q} - f_{0q}, \quad (16)$$

$$\phi_d = \phi_1 - \phi_0. \quad (17)$$

The phase-modulated portion is

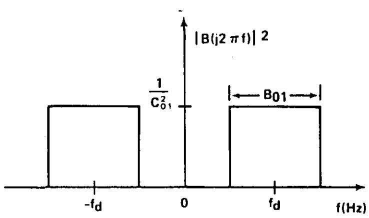

Fig. 3. Heterodyne receiver bandpass filter transfer function.

$$\dot{\theta}_d(t) = 2\pi K_m v_f(t) + \dot{\gamma}_d(t), \quad (18)$$

where

$$\gamma_d(t) = \gamma_1(t) - \gamma_0(t). \quad (19)$$

Using the quadrature component representation of the HR noise[6] and assuming that the noise spectrum is symmetric about the VCO quiescent frequency f_d, Eq. (12) can be rewritten as

$$v_d(t) = V_d \cos\Phi_d(t) + n_c(t)\cos(2\pi f_d t + \phi_d) + n_s(t)\sin(2\pi f_d t + \phi_d), \quad (20)$$

where $n_c(t)$ and $n_s(t)$ are the (statistically independent) in-phase and quadrature components of the noise $n(t)$, respectively. Equation (20) is an alternate form for the VCO output.

The reference and VCO signals, as seen in Fig. 2, meet in a phase detector (PD). The PD will be modeled as a multiplier with double frequency components removed.[7] The output of the PD, called the error signal, is then

$$v_e(t) = K_d L_p[v_r(t)v_d(t)], \quad (21)$$

where K_d is the PD gain, and $L_p[\ ]$ denotes the low-pass portion of the quantity in brackets. Substituting Eqs. (1) and (2) into the above equation results in

$$v_e(t) = [(K_d V_r V_d)/2][\sin\psi(t) + n'(t)], \quad (22)$$

where

$$n'(t) = [1/(V_d)][n_c(t)\sin\theta_r(t) + n_s(t)\cos\theta_r(t)], \quad (23)$$

$$\psi(t) = \Phi_r(t) - \Phi_d(t) = \theta_r(t) - \theta_d(t). \quad (24)$$

The variable $\psi(t)$ is called the loop phase error. As seen in the Eq. (23) the noise due to the HR is independent of the loop phase error, in contrast to classical PLL configurations.[8]

Referring again to Fig. 2, the error signal is now passed through a loop filter with impulse response $f(t)$, which gives a filtered error voltage of

$$v_f(t) = [(K_d V_r V_d)/2][\sin\psi(t) + n'(t)] * f(t), \quad (25)$$

where $*$ denotes convolution.

B. Loop Equation

With a preliminary discussion of the signals present in Fig. 2 now presented, we can complete the derivation of an equation for the total loop phase error. We note that the filtered error voltage is next fed back into the frequency modulator of the modulated laser changing its output frequency as described in Eq. (9). The instantaneous frequency of the VCO output is, from Eqs. (15) and (18),

$$\dot{\Phi}_d(t) = 2\pi[f_d + K_m v_f(t)] + \dot{\gamma}_d(t). \quad (26)$$

Substituting into this expression the results derived for filtered error signal, Eq. (25), gives

$$\dot{\Phi}_d(t) = 2\pi f_d + \{K[\sin\psi(t) + n'(t)] * f(t)\} + \dot{\gamma}_d(t), \quad (27)$$

where

$$K = \pi K_m K_d K_r K_d. \quad (28)$$

Finally, using Eq. (24), the nonlinear stochastic integro-differential equation for the loop phase error is

$$\dot{\psi}(t) = \dot{\theta}_r(t) - K[\sin\psi(t) + n'(t)] * f(t) - \dot{\gamma}_d(t). \quad (29)$$

This equation will be linearized by assuming that $\psi(t)$ is small (less than 1 rad) so that $\sin\psi(t) \approx \psi(t)$. Additionally, converting this equation into the frequency domain by taking the Laplace transform yields[8]

$$\Psi(s) = \Theta_r(s) - \Gamma_d(s) - \frac{KF(s)}{s}[\Psi(s) + N'(s)]. \quad (30)$$

(It is assumed that the transform of the sample functions of the random processes exists.) This linearized version of the integro-differential equation is graphically depicted in Fig. 4.

Of primary interest in the loop equation is the laser differential phase $\Phi_d(t)$. Since $\Phi_d(t) = 2\pi f_d t + \phi_d + \theta_d(t)$, it is sufficient to know only the modulated phase $\theta_d(t)$. Substituting $\Theta_r(s) - \Theta_d(s)$ for $\Psi(s)$ [from Eq. (24)] and solving for $\Theta_d(s)$ give

$$\Theta_d(s) = H(s)[N'(s) + \Theta_r(s)] + [1 - H(s)]\Gamma_d(s), \quad (31)$$

where

$$H(s) = \frac{KF(s)}{s + KF(s)}. \quad (32)$$

$H(s)$ is the closed-loop transfer function. From Eq. (3), the transform of the reference-modulated phase is

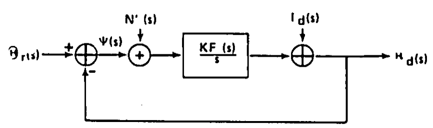

Fig. 4. Linear two-laser phase-lock loop model.

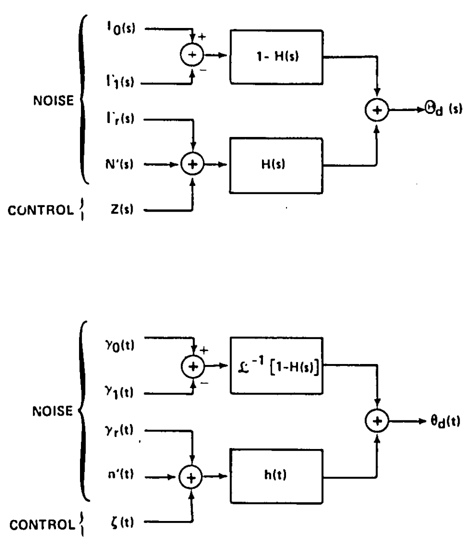

Fig. 5. Equivalent linear filter model of the two-laser phase-lock loop for (a) Laplace and (b) time domains.

$$\Theta_r(s) = Z(s) + \Gamma_r(s), \quad (33)$$

and from Eq. (19)

$$\Gamma_d(s) = \Gamma_1(s) - \Gamma_0(s). \quad (34)$$

Substituting these into the above gives

$$\Theta_d(s) = H(s)[N'(s) + Z(s) + \Gamma_r(s)] + [1 - H(s)][\Gamma_1(s) - \Gamma_0(s)]. \quad (35)$$

The laser differential phase error can thus be obtained by passing the various noise, instability, and control processes through a set of linear filters. Equation (35) is shown graphically in Fig. 5 for both Laplace and time domains. Solving Eq. (30) for $\Psi(s)$ yields

$$\Psi(s) = [1 - H(s)][\Theta_r(s) - \Gamma_d(s)] - H(s)N'(s). \quad (36)$$

This is the loop phase error between the laser differential phase and the desired reference phase. It will become important when considering the modulated steady-state response of the system in a later section.

With the loop response defined in terms of the loop equation, or equivalently a parallel set of filters, an evaluation of its performance can now be made. To simplify the problem, an equivalent model for $H(s)$, the closed-loop transfer function, will first be developed.

C. Noise Equivalent Bandwidth of the Closed-Loop Transfer Function

The closed-loop transfer function is from Eq. (32)

$$H(s) = \frac{KF(s)}{s + KF(s)}. \quad (32)$$

For any loop filter $F(s)$ considered here, we note that

$$|H(0)|^2 = 1. \quad (37)$$

The noise equivalent bandwidth (NEB) of $H(s)$ is defined as an ideal low-pass filter (LPF) with magnitude $|H(0)|^2$ and one-sided bandwidth W_H whose area under the curve is the same as the area under $|H(s)|^2$ across the entire spectrum.[8] That is,

$$W_H = \frac{1}{4\pi j}\int_{-j\infty}^{j\infty} |H(s)|^2 ds, \quad (38)$$

or in Fourier notation,

$$W_H = \int_0^{\infty} |H(j2\pi f)|^2 df. \quad (39)$$

This bandwidth is graphically depicted in Fig. 6.

The exact size of W_H will of course depend on the exact nature of $F(s)$. Although ideally $F(s)$ is only the loop filter which follows the PD in the loop (Fig. 2), this assumes that no filtering occurs elsewhere in the loop.

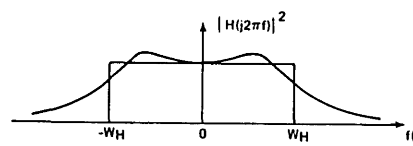

Fig. 6. Noise equivalent bandwidth model of the closed-loop transfer function.

However, realistically, $F(s)$ includes contributions not only from the designed loop filter, but from every other component in the loop (e.g., the response time of the laser phase modulator and the HR BPF); thus the component with the narrowest bandpass response will set the maximum limit on the value of W_H. In this model, the two dominant filtering sources are the designed filter and the HR BPF, the narrower of which will limit W_H.

An important measure of the steady-state performance of the phase-lock control loop is a second moment description of the laser differential phase $\theta_d(t)$. To find the mean and variance, the statistics of the noises and instabilities and the nature of the control modulation must be known. A first and second moment description of the noise and instabilities is thus discussed next.

III. Statistical Description of Loop Noise Sources

There are three sources of noise preventing an ideal steady value of laser differential phase. They are the loop reference phase instabilities, the individual laser phase instabilities, and the heterodyne receiver noise. A second moment model for each will be presented in this section.

A. Loop Input Reference Instabilities

As defined in Eq. (3) the phase modulation portion $\theta_r(t)$ of the loop input reference phase consists of a deterministic control modulation component $\zeta(t)$ and a random instability component $\gamma_r(t)$. It will be assumed that the phase instability is stationary, zero mean, with a constant power spectral density. Thus

$$E[\theta_r(t)] = \zeta(t), \tag{40}$$

$$S_{\theta_r}(f) = S_{\gamma_r}(f) = N_r. \tag{41}$$

B. Heterodyne Receiver Noise

The power spectral density of the output noise of an HR is well known and will simply be stated here. The results are conditioned on the previous assumptions made for the HR plus the following additional assumptions:

(1) For power or energy calculations only,

$$f_0 = f_{0q} \approx f_{1q}. \tag{42}$$

(2) There is negligible excess noise. (An excess noise term arises strictly from the random portion of the fields.[5]) With all the previous assumptions, the output of an HR is given by Eq. (12), where $n(t)$ is a zero-mean Gaussian random process with spectrum,[5]

$$S_n(f) = \frac{q^2\eta A_d A_0^2 R^2}{2hf_0C_{01}^2}, \qquad f_d - \frac{B_{01}}{2} \le |f| \le f_d + \frac{B_{01}}{2}$$
$$= 0, \text{ elsewhere.} \tag{43}$$

The variable B_{01} is the bandwidth of the HR bandpass filter and $1/(C_{01}^2)$ its magnitude. The other variables are as defined earlier (see Fig. 3).

It is convenient to use a narrowband noise model for HR noise [see Eq. (20)]. Since this spectrum is symmetric about f_d, the spectrum of the quadrature components of $n(t)$ is

$$S_{n_c}(f) = S_{n_s}(f) = \frac{q^2\eta A_d A_0^2 R^2}{hf_0C_{01}^2}, \qquad |f| \le \frac{B_{01}}{2}$$
$$= 0, \text{ elsewhere.} \tag{44}$$

The loop phase noise due to the HR is given by Eq. (23). Since $n_c(t)$ and $n_s(t)$ are the quadrature components of the zero mean process $n(t)$,

$$E[n(t)] = E[n_c(t)] = E[n_s(t)] = E[n'(t)] = 0. \tag{45}$$

Using Eq. (23) plus the fact that the cross-correlation between $n_c(t)$ and $n_s(t)$ is zero [since $S_n(f)$ is symmetric about f_d], we obtain the autocorrelation of $n'(t)$,

$$R_{n'}(t_2,t_1) \triangleq E[n'(t_2)n'(t_1)] \tag{46}$$

$$= [1/(V_d^2)]R_{n_c}(\tau)E[\cos\Delta\theta_r(\tau)], \tag{47}$$

where

$$\tau = t_2 - t_1 \tag{48}$$

$$\Delta\theta_r(\tau) = \theta_r(t_2) - \theta_r(t_1)$$
$$= \zeta(t_2) - \zeta(t_1) + \gamma_r(t_2) - \gamma_r(t_1). \tag{49}$$

This result is strictly not stationary. However, it will be assumed that the input reference is stable relative to the rest of the system so that $\gamma_r(t) \approx 0$. It is further assumed that the desired modulation $\zeta(t)$ will not change as fast as (will have a larger correlation time than) the other system processes. So for time intervals of interest it follows that[8]

$$\Delta\theta_r(\tau) \approx 0 \tag{50}$$

so that

$$R_{n'}(t_2,t_1) = R_{n'}(\tau) = [1/(V_d^2)]R_{n_c}(\tau) \tag{51}$$

or finally

$$S_{n'}(f) = [1/(V_d^2)]S_{n_c}(f), \tag{52}$$

which from Eqs. (44) and (13) reduces to

$$S_{n'}(f) = (hf_0)/(A_dA_1^2\eta), \qquad |f| \le [(B_{01})/2].$$
$$= 0 \text{ elsewhere.} \tag{53}$$

The power spectrum of the loop noise due to the HR is then proportional to the energy of a detected photon hf_0.

C. Laser Phase Instabilities

The last type of noise limiting the performance of the two-laser phase-lock loop is the instability of the phase

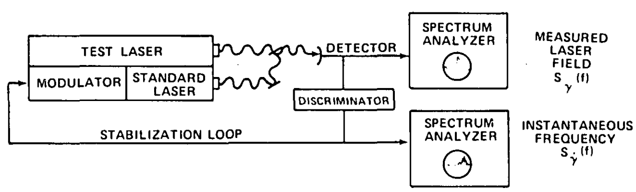

Fig. 7. Typical arrangement for measuring the spectra of a laser field and its instantaneous frequency.

of each laser. This section describes the power spectrum of laser phase in terms of the laser linewidth (which is measurable from the laser output field).

The output field of a laser can be represented by[1]

$$\mathcal{E}(t) = A \cos[2\pi f_0 t + \phi_0 + \gamma(t)], \tag{54}$$

where f_0 and ϕ_0 are, respectively, the constant frequency and phase of the field, and $\gamma(t)$ is a Gaussian random process. Arbitrarily, $\gamma(t)$ will be given a mean of zero by including any constant mean term in $(2\pi f_0 t + \phi_0)$.

An assumption about the stationarity of the phase instability $\gamma(t)$ is now made. Laser electric fields because of their inherently high frequencies are typically measured as a beat or relative field between two lasers as shown in Fig. 7.[9–11] For that type of configuration, where the measurements are made with respect to a standard laser whose instabilities are negligible compared with those of the laser to be tested, it has been shown that $\gamma(t)$ is a stationary Gaussian random process.[9–14] Such a measurement technique is assumed for all fields used in this paper. Thus, any statements herein regarding laser fields or spectra will be interpreted as the respective measured quantities. This interpretation is consistent with the types of measurements required by the two-laser phase-lock loop we are discussing.

Using the assumption that $\gamma(t)$ is a zero-mean, stationary, Gaussian random process, a relationship between the field spectrum and the phase instability has been derived.[9–15] The results will simply be stated here.

The random phase instability $\gamma(t)$ of lasers is composed of two components.[10] One is an external contribution $\gamma_e(t)$ due to acoustic noise, structural vibrations, plasma oscillations, thermal and pressure drifts, and other environmental disturbances. (It is also called technical noise[10] or extraneous modulation.[12,13]) The other element is a quantum contribution $\gamma_q(t)$, usually much weaker. Quantum noise is the quantum-mechanical or statistical limit of phase fluctuations and thus sets the minimum size of the phase spectrum.

Assuming that the form of the laser field spectrum due to quantum noise alone is Lorentzian with linewidth Δf_q, the spectrum of the phase fluctuation due to quantum noise is

$$S_{\gamma_q}(f) = (\Delta f_q)/(2\pi f^2). \tag{55}$$

The spectrum of the phase fluctuation due to external noise sources is

$$S_{\gamma_e}(f) = (f_c \sigma_e^2)/[\pi f^2 (f_c^2 + f^2)], \tag{56}$$

where f_c is an arbitrarily small cutoff frequency. This result assumes that the form of the laser field spectrum due to external noise alone is Gaussian in shape with a standard deviation (rms linewidth) of σ_e or equivalently a linewidth of Δf_e.

The observed field spectrum is the sum of the quantum and external spectrum results. Adding Eqs. (55) and (56) gives

$$S_\gamma(f) = S_{\gamma_q}(f) + S_{\gamma_e}(f) = \frac{1}{2\pi f^2}\left[\Delta f_q + \frac{2\sigma_e^2 f_c}{(f_c^2 + f^2)}\right]. \tag{57}$$

Equation (57) is the phase spectrum of a single laser, but as defined in Eq. (19), the laser differential phase instability $\gamma_d(t)$ is due to two lasers, the reference laser and the modulated laser. Since it was previously assumed that the laser instabilities are zero mean,

$$E[\gamma_d(t)] = 0. \tag{58}$$

It will further be assumed that the instabilities in each laser are statistically independent, which is reasonable if they are in their own thermally and acoustically isolated cavities. Using this assumption, the spectrum of the difference is the sum of the individual spectra. Thus from Eq. (57) we obtained

$$S_{\gamma d}(f) = \frac{1}{f^2}\left[(\alpha_0 + \alpha_1) + \frac{\beta_0 f_{c0}}{f_{c0}^2 + f^2} + \frac{\beta_1 f_{c1}}{f_{c1}^2 + f^2}\right], \tag{59}$$

where

$$\alpha = (\Delta f_q)/2\pi \tag{60}$$

$$\beta = \frac{\sigma_e{}^2}{\pi} = \frac{\Delta f_e^2}{8\pi \ln 2}. \tag{61}$$

With the statistical nature of the reference instability, the lasers's instabilities, and the HR noise now specified, the performance of the laser differential phase from Eq. (35) can now be determined.

IV. Loop Performance

The performance of the control loop will be measured in terms of the steady-state (SS) mean and variance of the laser differential phase $\Phi_d(t)$. This will be done first with no modulation, that is, $\zeta(t) = 0$. For the no-modulation case the noise equivalent bandwidth

$$W_{H_{\min}},$$

which results in the minimum value of phase variance, will be determined. Next the SS mean will be determined in the modulated case for different modulation schemes and filter types. The transient system response for the modulated case will also be examined.

A. No Modulation: Steady-State Mean and Variance

The laser differential phase error is given by Eq. (35). Using the expression for the expected value of laser phase noise [Eq. (58)], HR noise [Eq. (45)], loop input reference phase modulation [Eq. (40)], and the fact that $\zeta(t) = 0$ (no modulation), the mean of the laser differential phase error is

$$E[\theta_d(t)] = E[\Theta_d(s)] = 0. \tag{62}$$

Thus the constant phase between the lasers from Eqs. (2) and (15) is

$$E[\Phi_d(t)] = E[\Phi_r(t)] = 2\pi f_d t + \phi_d. \tag{63}$$

That is, when the system is unmodulated, the phase difference between the two lasers is equal to the loop reference phase. This, of course, is the desired result.

The variance is a measure of fluctuations away from the desired mean. Equation (35) shows that the phase

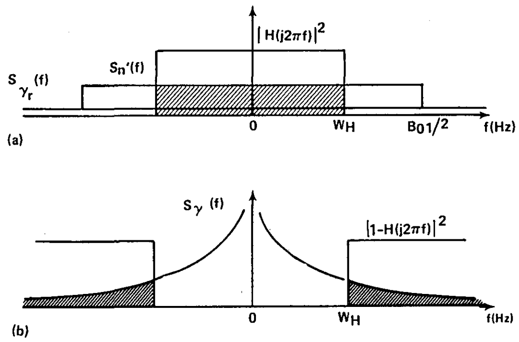

Fig. 8. Graphical representation of contributions to the laser differential phase error variance due to (a) the heterodyne receiver and input reference noise and (b) the laser's phase instabilities.

error is obtained by passing HR noise and reference phase noise through the filter $H(s)$ and the laser instabilities through the filter $[1 - H(s)]$. The reference modulation in this case is $\zeta(t) = 0$ [thus $Z(s) = 0$]. Using the NEB of $H(s)$ derived earlier and assuming that all the noise processes are statistically independent, the variance of the phase error is

$$\sigma^2 = 2\int_0^{W_H} [S_{n'}(f) + S_{\gamma_r}(f)]df + 2\int_{W_H}^{\infty} S_{\gamma d}(f)df. \qquad (64)$$

The two terms of this integral are illustrated in Fig. 8 to aid in showing the effect of W_H on the resulting σ^2. The total cross-hatched area on the two graphs is the variance. In Fig. 8(a), as previously noted, $B_{01}/2$ can never be less than W_H. Thus from the point of view of the closed-loop transfer function, the HR noise $n'(t)$ is nearly white.

Using the values for the spectral densities of Eqs. (41), (53), and (59) and integrating reduces Eq. (64) to

$$\sigma^2 = 2N_r W_H + \frac{2hf_0W_H}{\eta A_d A_1^2} + \frac{2(\alpha_0 + \alpha_1)}{W_H} + \frac{2\beta_1}{f_{c1}^2}\left[\frac{f_{c1}}{W_H} - \frac{\pi}{2} + \tan^{-1}\left(\frac{W_H}{f_{c1}}\right)\right] + \frac{2\beta_0}{f_{c0}^2}\left[\frac{f_{c0}}{W_H} - \frac{\pi}{2} + \tan^{-1}\left(\frac{W_H}{f_{c0}}\right)\right]. \qquad (65)$$

From the section on laser phase instabilities, f_{c0} and f_{c1} are arbitrarily small (experimentally less than 1 Hz[9]), so it is reasonable to assume that

$$W_H/f_{c0} \gg 1; \qquad W_H/f_{c1} \gg 1. \qquad (66)$$

Using that result, the $\tan^{-1}$ terms can be approximated by the first three terms of their power series as

$$\tan^{-1}\frac{W_H}{f_c} \approx \frac{\pi}{2} - \frac{f_c}{W_H} + \frac{f_c^2}{3W_H^2}. \qquad (67)$$

Substituting this approximation back into Eq. (65) finally yields an expression for the laser differential phase error variance of

$$\sigma^2 = C_1W_H + \frac{C_2}{W_H} + \frac{C_3}{W_H^2}, \qquad (68)$$

where

$$C_1 = [(2hf_0)/(\eta A_d A_1^2)] + 2N_r, \qquad (69)$$

$$C_2 = 2(\alpha_0 + \alpha_1), \qquad (70)$$

$$C_3 = [2(\beta_0 + \beta_1)]/3. \qquad (71)$$

The variable C_1 is proportional to the energy of a detected photon hf_0; C_2 is proportional to Δf_q, the sum of the quantum-limited laser field linewidths; and C_3 is proportional to Δf_e, the sum of the laser field linewidths due to external effects. The conflicting effects of the closed-loop bandwidth of the terms in the phase error variance indicate that some optimization of W_H is desirable.

To optimize the loop SS performance, the variance of the laser differential phase must be minimized. The expression for phase error variance is given by Eq. (68), and a plot showing the effect of the three different terms is given in Fig. 9. To find W_H such that σ^2 is minimal, we solve for the stationary point of Eq. (68) with respect to W_H, yielding

$$W_{H_{\min}} = \left\{\frac{C_3}{C_1} + \left[\left(\frac{C_3}{C_1}\right)^2 - \left(\frac{C_2}{3C_3}\right)^3\right]^{1/2}\right\}^{1/3} + \left\{\frac{C_3}{C_1} - \left[\left(\frac{C_3}{C_1}\right)^2 - \left(\frac{C_2}{3C_1}\right)^3\right]^{1/2}\right\}^{1/3}. \qquad (72)$$

If $C_3^2 \gg C_2^3$, which is typically the case, since for most lasers $\Delta f_e \gg \Delta f_q$,[8] Eq. (72) simplifies to

$$W_{H_{\min}} \approx \left(\frac{2C_3}{C_1}\right)^{1/3}. \qquad (73)$$

It is obvious from the examination of Fig. 9 that if the actual bandwidth is less than $W_{H_{\min}}$, the variance is limited by the laser phase instabilities (i.e., laser phase noise dominates). If W_H is greater than $W_{H_{\min}}$, the variance is dominated by detector noise and input reference instabilities. It will be shown that in many conceivable systems,

$$W_H < W_{H_{\min}}$$

because $W_{H_{\min}}$ is very large, and thus the variance is limited by the phase instabilities of the lasers and is independent of the HR measurement noise (see the numerical example in a later section).

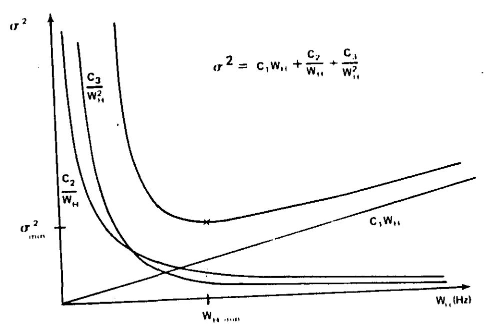

Fig. 9. Closed-loop bandwidth vs laser differential phase error variance.

Table I. Basic Parameters for First-, Second-, and Third-Order Loop Filters

Loop order	Loop filter transfer function $F(s)$	Closed-loop transfer function $H(s)$	$1 - H(s)$	Noise equivalent bandwidth W_H (Hz)
1	1	$\frac{K}{s+K}$	$\frac{s}{s+K}$	$\frac{K}{4}$
2	$1+\frac{a}{s}$	$\frac{K(s+a)}{s^2+K(s+a)}$	$\frac{s^2}{s^2+K(s+a)}$	$\frac{K+a}{4}$
3	$1+\frac{a}{s}+\frac{b}{s^2}$	$\frac{K(s^2+as+b)}{s^3+K(s^2+as+b)}$	$\frac{s^3}{s^3+K(s^2+as+b)}$	$\frac{K(aK+a^2-b)}{4(aK-b)}$

Table II. Control Modulation Schemes in Temporal and Laplace Representation

Modulation scheme	Time domain	Laplace representation
Step in phase	$\Delta\phi u(t)$	$\frac{\Delta\phi}{s}$
Step in frequency (ramp in phase)	$2\pi\Delta f t u(t)$	$\frac{2\pi\Delta f}{s^2}$
Ramp in frequency (phase acceleration)	$\frac{Dt^2}{2}u(t)$	$\frac{D}{s^3}$

B. Modulated Reference Signals

The real power of a phase-lock control loop is its ability to track a modulated reference input. The loop filter $F(s)$, the principal determinant of the loop's modulated performance, is typically optimized for the type of modulation desired. This section examines the performance of the loop for some specified modulation schemes and filter types.

As stated earlier, $F(s)$ in Fig. 2 actually includes all filtering by any component in the loop. It will be assumed that all these components are controllable, and thus $F(s)$ can be constructed as desired.

There are several loop filters that are commonly used in phase-lock loops.[16] These are the first-, second-, and third-order filters and are listed in Table I. The expressions for $H(s)$ and W_H were calculated using Eqs. (32) and (39), respectively.

The modulation schemes to be examined are listed in Table II in both Laplace and time domains. (Since the modulation schemes are causal, they are unique transform pairs.) The variable $\zeta(t)$ from Eq. (3) is the control phase modulation of the loop reference phase and is in units of radians.

Once the filter and modulation scheme are known, the mean steady-state response for the laser differential phase error can be determined. For this determination, the loop phase error $\psi(t)$ becomes important. From Eq. (24),

$$\psi(t) = \theta_r(t) - \theta_d(t), \tag{24}$$

and using Eq. (40) [and the assumption that $\theta_r(t)$ and $\theta_d(t)$ are statistically independent] and exchanging terms gives

$$E[\theta_d(t)] = \zeta(t) - E[\psi(t)]. \tag{74}$$

Thus the mean steady-state value for the laser differential phase error can be found by knowing the mean steady-state value for the loop phase error and the control modulation of the reference phase. For instance, if $\zeta(t)$ is a ramp in frequency $(Dt^2)/2$, and the loop is second order, using Eq. (36) and the final value theorem, the mean steady-state loop phase is

$$\lim_{t\to\infty} E[\psi(t)] = \lim_{s\to 0} sE[\Psi(s)] = \lim_{s\to 0} s\left(\frac{s^2}{s^2+Ks+aK}\right)\frac{D}{s^3}, \tag{75}$$

$$E[\psi(t)]_{ss} = D/aK \quad \text{(rad)}. \tag{76}$$

Now utilizing Eq. (74) the mean steady-state laser differential phase error is

$$E[\theta_d(t)]_{ss} = [(Dt^2)/2] - [D/(aK)]. \tag{77}$$

That is, the loop will follow the frequency ramp (accelerating phase), but there will be a constant phase error offset of D/aK rad between the two lasers. This procedure is followed for all combinations of filter orders and reference phase modulations schemes, and the results are summarized in Table III. Only the third-order loop will follow accelerating phase with no constant phase offset. (The first-order loop has a phase and frequency offset when trying to track accelerating phase and is not calculated.)

C. Dynamic Response

The loop filter, as shown previously, is important in determining the loop performance. First, the filter determines the closed-loop transfer function and resulting loop bandwidth (Table I). This in turn influences the steady-state variance of the laser differential phase error caused by the laser and input reference instabilities and detector noise. Second, the filter determines the steady-state response of the loop to various modulation patterns (Table III). In addition to these steady-state performances, the loop filter also determines the transient or dynamic response of the loop.

The most versatile filter, as seen in Table III, is the one resulting in a third-order loop. The exact behavior of a third-order loop is complex and difficult to obtain.[16] However, it can be roughly contrasted with the second-order loop whose behavior is relatively well known.

When a third-order loop is not initially locked to the loop reference phase, it is not as stable in its ability to acquire lock as the second-order loop. The second-order results could in this case act as an upper bound (best case) on the third-order loop performance.[17] Once locked, however, the third-order can outperform the second-order loop, most notably in its ability to follow accelerating phase (frequency ramp) with no steady-state phase error. In this case, second-order loop performance acts as a rough lower bound on third-order loop performance. More specific results must be obtained by computer simulation.

Some important performance equations for a second-order loop in a noiseless environment are listed in Table IV. Since they are valid for a noiseless environment, they represent the best possible performance in a noisy environment.

Table III. Mean Steady-State Value of the Laser Differential Phase for Different Loop Orders and Modulation Schemes

$E[\theta_d(t)]_{ss}$ (rads)	$\zeta(t)$ (rads)		
Loop orders	$\Delta\phi u(t)$	$2\pi\Delta f t u(t)$	$\frac{Dt^2}{2}u(t)$
1	$\Delta\phi$	$2\pi\Delta f t - \frac{2\pi\Delta f}{K}$	—[a]
2	$\Delta\phi$	$2\pi\Delta f t$	$\frac{Dt^2}{2} - \frac{D}{aK}$
3	$\Delta\phi$	$2\pi\Delta f t$	$\frac{Dt^2}{2}$

[a] The first-order loop has a phase and frequency offset when trying to track accelerating phase and is not calculated.

These results are in terms of radian frequency ω (rad/sec) instead of f (Hz), since ω is typically used in the literature when dealing with dynamic feedback system behavior. (The general relationship is $\omega = 2\pi f$.) Some results utilize two loop-parameter dependent variables called the natural frequency ω_n and the damping factor ξ. They are defined as follows[17]:

$$\omega_n = (aK)^{1/2}, \tag{78}$$

$$\xi = \tfrac{1}{2}(K/a)^{1/2} = [K/(2\omega_n)]. \tag{79}$$

The natural frequency is the system phase error oscillation frequency in response to a step in the input reference frequency when there is no damping ($\xi = 0$). The damping factor is a measure of how quickly the oscillations attenuate as the loop acquires lock. Many of the expressions for the same parameter vary depending on the reference, so all sources are also listed in the table.

Two two main areas of transient behavior are tracking and acquisition. Tracking parameters assume that the loop is initially locked to the input reference phase. The hold-in range $\Delta\omega_H$ is the maximum amount the reference frequency can deviate from the VCO quiescent frequency (HR output frequency) and have the loop remain locked. It is theoretically infinite for both second- and third-order loops.[17] Another important tracking parameter is the frequency step limit $\Delta\omega_s$. It is the maximum step in reference frequency below which the loop will not skip cycles. Cycle skipping occurs when the difference between the reference and VCO frequencies is large enough to create a differential frequency (beat frequency) at the PD output. Finally, the maximum reference frequency ramp that the loop will follow without dropping out of lock is called the maximum tracking sweep rate $D_{\max}$ (rad/sec^2).

Table IV. Equations For Dynamic Performance of a Second-Order Phase-Lock Loop in the Absence of Noise

	Tracking		
Frequency step limit	$\Delta\omega_s = 1.8\omega_n(\xi + 1)$	$\frac{\text{rad}}{\text{sec}}$	Ref. 7
Hold-in range	$\Delta\omega_H = \pm\infty$	$\frac{\text{rad}}{\text{sec}}$	Ref. 17
Maximum tracking sweep rate	$D_{\max} = \omega_n^2$	$\frac{\text{rad}}{\text{sec}^2}$	Refs. 7, 16, 17
	$= 2\omega_n^2$ (third-order loop)		Refs. 16, 17
	Acquisition		
Lock-in frequency	$\Delta\omega_L = 2\xi\omega_n$	$\frac{\text{rad}}{\text{sec}}$	Ref. 17
	$= 2\omega_n(\xi + 0.6)$		Ref. 17
Lock-in time	$T_L = 1.5/W_H$	sec	Ref. 17
	$= \pi/\omega_n$		Ref. 7
Pull-in frequency	$\Delta\omega_p = \pm\infty$	$\frac{\text{rad}}{\text{sec}}$	Refs. 16, 17
Pull-in time	$T_p = (\Delta\omega)^2/2\xi\omega_n^3$	sec	Refs. 7, 17
Maximum acquisition sweep rate	$R_{\max} = \omega_n^2/2$	$\frac{\text{rad}}{\text{sec}^2}$	Refs. 7, 17
	$= a(4W_H - a)$		Ref. 16

Acquisition parameters assume that the loop is not initially locked. The pull-in frequency $\Delta\omega_p$ is the maximum amount the reference frequency can deviate from the VCO quiescent frequency and still permit the loop to converge to a locked state. This, like the hold-in range, is also theoretically infinite. As the loop converges on the reference frequency, the VCO frequency finally gets close enough to the reference frequency so that the loop stops skipping cycles and settles into the locked state. The range within which the loop ceases skipping cycles after pull-in is called the lock-in frequency $\Delta\omega_L$. The associated times for the loop to pull-in and lock-in to a step change in reference frequency are, respectively, T_p and T_L. The pull-in time T_p includes only the time that the loop cycle-skips. The lock-in time T_L is the time from the last cycle-skip until lock is acquired. The tracking frequency step limit $\Delta\omega_s$ is often equated with the acquisition lock-in frequency $\Delta\omega_L$, since the basic criterion in the definition of each is the maximum frequency difference below which the loop does not skip cycles. It does not matter if this frequency difference occurs as a result of an input reference frequency step with the loop initially locked or as a result of pull-in with the loop attempting to acquire lock. This can readily be seen in the similarity of the expressions for $\Delta\omega_s$ and $\Delta\omega_L$ in Table IV. The lock-in time T_L thus applies to either case. Finally, the maximum frequency ramp that the loop can sweep and still acquire lock is called the maximum acquisition sweep rate $R_{\max}$ (rad/sec^2).

It is important to note that these transient performance criteria may dominate simple noise considerations. If, for example, W_H was chosen to minimize the phase error variance [Eq. (72)], it might not be large enough to meet, say, the acquisition sweep rate demanded of the system.

The steady-state mean and variance of $\Phi_d(t)$ due to unwanted fluctuations (detector noise, laser instabilities, and reference instabilities) has been examined when the loop input reference was not modulated. The steady-state and dynamic changes in $\Phi_d(t)$ with various modulation schemes and filter designs were also examined. The effect of any given individual parameter [e.g., $F(s)$, $\gamma_d(t)$, $n(t)$, $\zeta(t)$] on the over-all laser differential phase $\Phi_d(t)$ [or equivalently the modulated portion $\theta_d(t)$] can most readily be understood by examination of Fig. 5 [Eq. (35)]. A design example for a potentially realizable two-laser control system will now be presented to illustrate the effects of the various parameters on the performance of the loop.

V. Numerical Design Example

The specific system to be examined here pairs a highly stable CO_2 reference laser with a highly adjustable CO_2 waveguide laser. CO_2 lasers were chosen because of their widespread use. The waveguide laser was chosen because it has a very wide gain bandwidth and thus can operate over a wide range of frequencies around the centerline frequency.[18]

The transition of interest in this example is $\lambda = 10.6$ μm, which corresponds to

$$f_0 = 2.83 \times 10^{13}\ \text{Hz}. \tag{80}$$

The reference laser has the following characteristics:

$$\Delta f_{q0} = 6.28 \times 10^{-4}\ \text{Hz}; \tag{81}$$

$$\Delta f_{e0} = 7.77 \times 10^{3}\ \text{Hz}. \tag{82}$$

The waveguide laser is assumed to have the following characteristics:

$$\Delta f_{q1} = 2.00 \times 10^{-5}\ \text{Hz}; \tag{83}$$

$$\Delta f_{e1} = 2.12 \times 10^{7}\ \text{Hz}. \tag{84}$$

Also, it has a usable gain linewidth (range over which the output frequency is adjustable and the laser power out does not drop below half its maximum value) of 7.00×10^8 Hz. As is true for most lasers, the linewidths of both lasers in this example are dominated by external disturbances, not quantum limitations. With the quantum and external linewidths for both lasers known, Eqs. (60), (61), (70), and (71) can be used to find

$$C_2 = 2.06 \times 10^{-4}\ \text{sec}^{-1}, \tag{85}$$

$$C_3 = 1.72 \times 10^{13}\ \text{sec}^{-2}. \tag{86}$$

Clearly, Δf_{e1} dominates in the term C_3. The HR samples 1×10^{-3} W ($= A_d A_1^2$) and has a quantum efficiency of $\eta = 0.5$. The reference is assumed to be very stable so that $N_r \approx 0$. Using Eq. (69) it is found that

$$C_1 = 7.51 \times 10^{-17}\ \text{sec}. \tag{87}$$

If the HR BPF is designed to allow the frequency to range over the entire usable gain bandwidth, B_{01} must be greater than 7.00×10^8 Hz. Arbitrarily, B_{01} is picked to be equal to that amount:

$$B_{01} = 7.00 \times 10^{8}\ \text{Hz}. \tag{88}$$

If it is assumed that the loop filter does not limit the bandpass of the loop, W_H is at most $B_{01}/2$. The NEB W_H of the closed-loop transfer function reasonably must be somewhat less than that and is arbitrarily assumed to be

$$W_H = 4.15 \times 10^{7}\ \text{Hz}. \tag{89}$$

Using the results for C_1, C_2, C_3, and W_H the laser differential phase variance from Eq. (68) is

$$\sigma^2 = 9.99 \times 10^{-3}\ \text{rad}^2 \tag{90}$$

or standard deviation of

$$\sigma = 0.100\ \text{rad} = 5.75\ \text{deg}. \tag{91}$$

This is fairly good performance. However, if W_H is decreased by only 1 order of magnitude to 4.15×10^6 Hz, the standard phase deviation will increase to $\sigma = 1$ rad (57.3 deg). The minimum variance bandwidth using Eq. (73) is

$$W_{H_{\min}} = 7.7 \times 10^{9}\ \text{Hz}. \tag{92}$$

This is more than 2 orders of magnitude greater than the actual W_H. Thus the system is dominated by laser phase noise. The variance at

$$W_{H_{\min}}$$

is

$$\sigma_{\min}^2 = 8.68 \times 10^{-7}\ \text{rad}^2 \tag{93}$$

or

$$\sigma_{\min} = 0.932\ \text{mrad}\ (0.053\ \text{deg}), \tag{94}$$

which is the best noise variance this particular loop can attain.

A second-order loop will be used in this example. A commonly employed compromise between stability and speed of response for a second-order system is to let[16]

$$\xi = 0.707. \tag{95}$$

Using the expression for W_H of a second-order loop from Table I and Eq. (79) to solve for K gives

$$K = 1.11 \times 10^8\ \text{sec}^{-1}. \tag{96}$$

Again using Eq. (79) gives

$$a = 5.53 \times 10^7\ \text{sec}^{-1}, \tag{97}$$

and Eq. (78) results in

$$\omega_n = 7.83 \times 10^7\ \text{rad/sec}. \tag{98}$$

The transient performance of the loop is found by using the results of Table IV. The tracking frequency step limit is

$$\Delta\omega_s = 2.41 \times 10^8\ \text{rad/sec} = 3.83 \times 10^7\ \text{Hz}. \tag{99}$$

The maximum tracking sweep rate is

$$D_{\max} = 6.13 \times 10^{15}\ \text{rad/sec}^2 = 9.76 \times 10^{14}\ \text{sec}^{-2}, \tag{100}$$

while the acquisition lock-in frequency is

$$\Delta\omega_L = 2.05 \times 10^8\ \text{rad/sec} = 3.26 \times 10^7\ \text{Hz}. \tag{101}$$

As long as the changes in the reference input frequency are less than $\Delta\omega_L$ ($\approx\Delta\omega_s$), the loop will have a lock-in time of no more than

$$T_L = 40\ \text{nsec}. \tag{102}$$

If, however, the reference frequency changes by the full amount of usable laser gain linewidth, 7.00×10^8 Hz, the pull-in time is

$$T_p = 722\ \text{nsec}, \tag{103}$$

which is in addition to the lock-in time, giving a maximum time to lock of 762 nsec for the system. This is very useful and could allow modulation on the order of milliseconds with the loop still in lock. The maximum acquisition sweep rate is

$$R_{\max} = 3.07 \times 10^{15}\ \text{rad/sec}^2 = 4.88 \times 10^{14}\ \text{sec}^{-2}.$$

There is, however, a steady-state phase error if this second-order loop tracks a frequency ramp. If, for example, the tracking sweep rate is at its maximum allowable value for the loop ($D_{\max} = 6.13 \times 10^{15}$ rad/sec^2), the steady-state phase error from Table III is 1 rad (57.3 deg). A third-order loop must be used to eliminate this error.

This example has shown that even if one laser of the pair has a large field spectrum linewidth ($\sim 2.1 \times 10^7$ Hz), the loop can perform very well, with a standard deviation between the two-laser phases of less than 6 deg and a maximum locking time of about 762 nsec. It is important to note that the system performance is limited by the laser phase instabilities (expecially that of the waveguide laser) and not by detector noise. The key elements in improving performance are to make W_H as large as possible and to make the lasers as stable as possible.

VI. Conclusions

The basic result of this paper, given by Eq. (68), is the fundamental limitation in performance, in terms of the variance of the differential phase, of a PLL designed to set the differential phase between the output fields of two lasers to a desired value. The fundamental noise sources are the HR noise and the laser phase instabilities, which are oppositely affected by the closed-loop bandwidth. As the example showed, the bandwidth must be extremely large before HR noise becomes the dominant noise source. Thus, in most cases, it is likely that the laser phase instabilities will ultimately limit loop performance. Other measures of performance when the loop is modulated were also discussed.

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