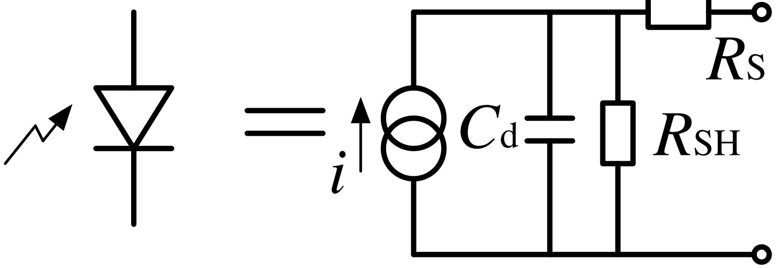
A) Equivalent circuit for a photodiode

K. Arai, Dec 5th, 2010



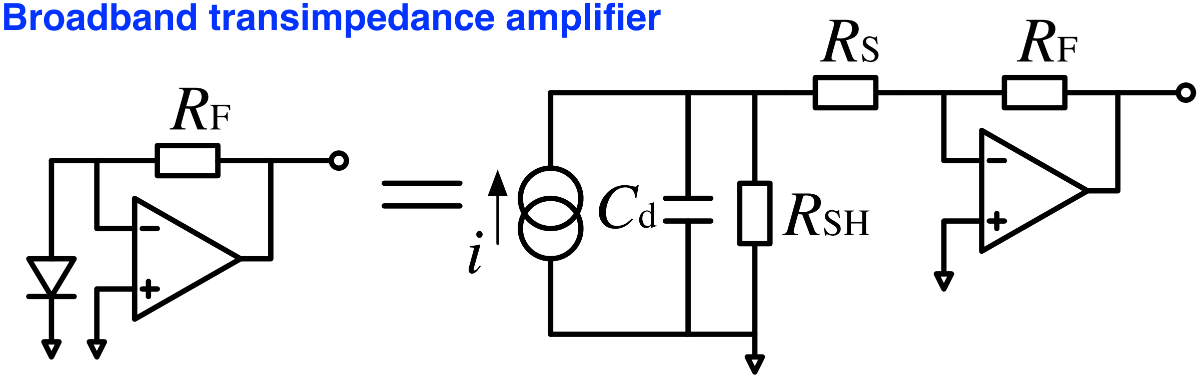
$R_{SH} = 4\,k\,T/i_{dark}^2$

~100M Ohm for $1.3\text{x}10^{-14}\,\text{A/Hz}^{1/2}$

$R_S \sim$ 10 Ohm

$C_d \sim$ 100pF

B) Broadband transimpedance amplifier



$V_{shot} = i_{shot}\, R_F = R_F\, \text{sqrt}(2\ \text{e}\ i_{dc})$

$V_{th,RSH} = R_F\, \text{sqrt}[4\, k\, T\, /\, R_{SH}]$

$V_{th,RF} = \text{sqrt}[4\, k\, T\, R_F]\ (>> V_{th,RSH})$

$V_{th,RS} \sim R_F\, \text{sqrt}[4\, k\, T\, R_S]\ /\ R_{SH} * (1+\text{i}\, f\, /\, f_{SH})\ /\ (1+\text{i}\, f\, /\, f_S)$

$(\ f_{SH}=1/(2\ \text{pi}\ R_{SH}\ C_d),\ f_S=1/(2\ \text{pi}\ R_S\ C_d)\ )$

$V_{th,RS@f=0} = R_F\, \text{sqrt}[4\, k\, T\, R_S]\ /\ R_{SH}$

$V_{th,RS@f=High} = R_F\, \text{sqrt}[4\, k\, T\, /\, R_S]$

$V_{shot} = R_F \,\mathrm{sqrt}(2\, e\, i_{dc})$

Low freq regime

Amplifier noise is dominant (unless $R_{SH} \sim R_F$)

$V_{th} \sim \mathrm{sqrt}[4\, k\, T\, R_F]$

$V_{shot} = V_{th}$ when

$i_{dc} = 2\, k\, T /(e\, R_F)$

e.g. $i_{dc} = 0.05$mA for $R_F = 1000$ Ohm ==> ($V_{th} = 4\mathrm{nV/Hz}^{1/2}$)

High freq regime

Noise of R_S is dominant

(i.e. current noise of R_S, or voltage noise of R_S amplified by R_F/R_S)

R_{SH} can be ignored

$V_{th} \sim R_F \,\mathrm{sqrt}[4\, k\, T / R_S]\, |(i\, f / f_S) / (1+i\, f / f_S)|$

$V_{shot} = V_{th}$ when

$i_{dc} = 2\, k\, T /(e\, R_S)\, |(i\, f / f_S) / (1+i\, f / f_S)|^2$

With the example parameters: f_S=160MHz

most of the case $f < f_S$

$i_{dc} = 2\,k\,T/(e\,R_S) * (f/f_S)^2 \quad = \; 5\,(f\ /\ 160\text{MHz})^2$ [mA]

$V_{th} \sim R_F\,(f/f_S) * \text{sqrt}[4\,k\,T/R_S] \; = \; 40\,(f\ /\ 160\text{MHz})$ [nV/Hz$^{1/2}$]

i.e. The noise of R_F is dominant at f<16MHz

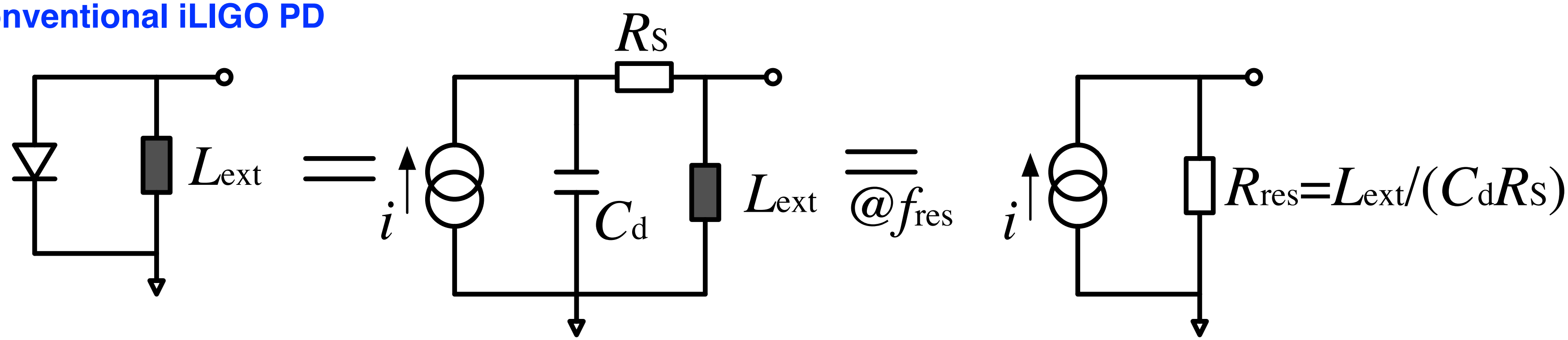
Also note that the bandwidth of the PD is limited by $C_d\,R_F$

$f_c = \text{sqrt}[\text{GBW} / (2\,\text{pi}\,C_d\,R_F)]$

e.g. GBW=1GHz, C_d=100pF, R_F=1000 Ohm ==> $f_c \sim$ 40MHz

http://lhocds.ligo-wa.caltech.edu:8000/40m/40m_Library?action=AttachFile&do=get&target=PD_TF_20011019.pdf

C) conventional iLIGO PD



Apparently we can ignore shunt regulator RSH

$$V_{shot} = \mathrm{sqrt}(2\ e\ i_{dc})\ R_{res}$$
$$V_{th} = \mathrm{sqrt}(4\ k\ T\ R_{res})$$

$V_{shot}=V_{th}$ when

$$i_{dc} = 2\ k\ T\ /(e\ R_{RES})$$
$$= 2\ k\ T\ C_d\ R_S\ /(e\ L_{ext})$$

$$i_{dc} = 2\ k\ T\ C_d\ R_S\ /(e\ L_{ext}) = 2\ k\ T\ /(e\ R_S) * (f_{res}/f_S)^2$$

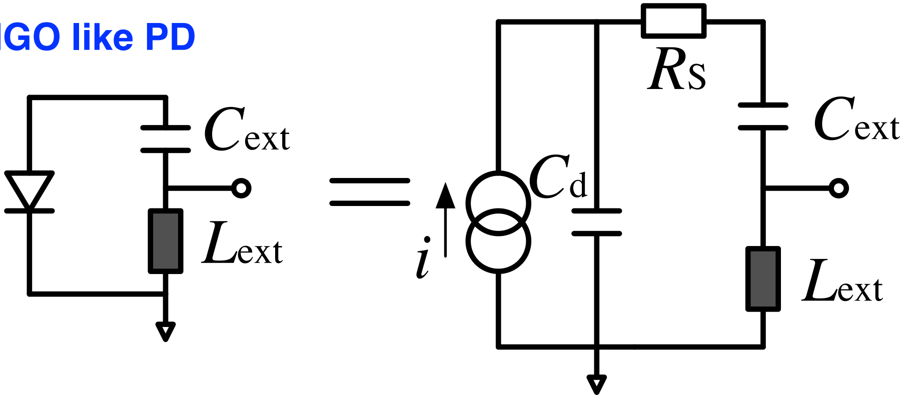
Equivaent to the broadband case!

$$V_{th} = \mathrm{sqrt}(4\ k\ T\ R_{res}) = R_S\ (f_S/f_{res}) * \mathrm{sqrt}(4\ k\ T\ /\ R_S)$$

Amplifier requirement

resonant vs broadband: $R_S\ (f_S/f) > (f/f_S)$ when $f < 500$MHz

D) aLIGO like PD



Again the shunt regulator is ignored.
Rs may include the series resistor of the external capacitor.

Resonant condition: $L_{ext} = (1/C_d + 1/C_{ext})/(2\,\mathrm{pi}\, f_{res})^2$

$V_{shot} = \mathrm{sqrt}(2\,e\,i_{dc})\,R_S\,(f_S + f_{ext}) f_S/(f_{res})^2$

$V_{th} = (f_S + f_{ext})/f_{res}\,\mathrm{sqrt}(4\,k\,T\,R_S)$ Slightly larger than c)

$(f_S = 1/(2\,\mathrm{pi}\,R_S\,C_d),\ f_{ext} = 1/(2\,\mathrm{pi}\,R_S\,C_{ext}))$

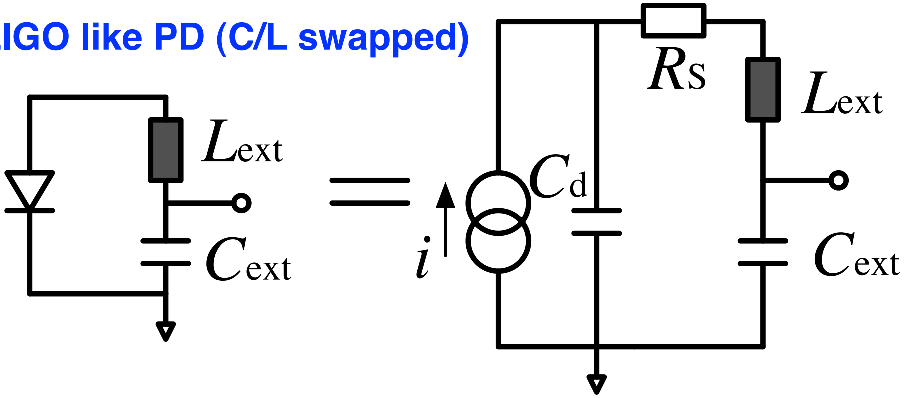
$V_{shot} = V_{th}$ when

$i_{dc} = 2\,k\,T/(e\,R_S) * (f_{res}/f_S)^2$

Equivalent to the iLIGO PD case!

The cut off freqs f_S and f_{ext} higher, the amplification of the signal larger (i.e. easier to work with)

D') aLIGO like PD (C/L swapped)



Resonant condition: $L_{ext} = (1/C_d + 1/C_{ext})/(2\,\mathrm{pi}\, f_{res})^2$ Same as D)

$V_{shot} = \mathrm{sqrt}(2\, e\, i_{dc})\, R_S\, (f_S\, f_{ext})/(f_{res})^2$ Smaller than D)

$V_{th} = (f_S + f_{ext})/f_{res}\ \mathrm{sqrt}(4\, k\, T\, R_S)$ Same as D)

$V_{shot} = V_{th}$ when

$i_{dc} = 2\, k\, T\, /(e\, R_S) * (f_{res}/f_S + f_{res}/f_{ext})^2$

Worse than D) case

In general the performance is worse than the case D)
owing to the smaller amplification of the signal