Keywords: lna, rf, rfic, amplifier, stability, power gain, transmission lines, rfics, theory, smith chart, rf ics, radio frequency, amplifiers, low noise amplifier, amps

Jan 09, 2003

APPLICATION NOTE 1849

Low-Noise Amplifier Stability Concept to Practical Considerations, Part 1

Abstract: Part one of a three-part series. Presents a brief overview of transmission lines and power gain definitions. Provides the basic background needed to design amplifiers for stability.

- Part 1
- Part 2
- Part 3

The design of small-signal, low-noise RF amplifiers is a step-by-step logical procedure with an exact solution for each problem. To aid the designer, many books offer complete schematics with component values "adaptable to the needs of a given application." But such circuits are designed for a specific set of operating conditions that may not correspond to your requirements. The absence of design procedures in these texts can leave readers helpless when attempting to adapt their circuit to a particular set of operating conditions.

This article takes the opposite approach. It presents a design process in which detailed, step-by-step procedures allow you to choose the LNA you want and operate it under any realistic operating conditions. You no longer have to adapt someone else's schematic to your specifications. Instead, you can create your own RF low-noise amplifiers and optimize them for a targeted application.

This article (Part 1) begins the discussion with a brief overview of transmission lines and a reminder on RF power gain definitions.

In Part 2, we jump into the RF aspect of low-noise amplifiers by examining stability (the tendency for oscillation), impedance matching, and general amplifier design, using the scattering parameters (s-parameters) as design tools.

Part 3 completes the series by presenting application examples. The first shows how to match an LNA in the maximum available gain condition. The second deals with an LNA matched in the constant desired gain condition. The third exercise stresses the importance of matching a potentially unstable LNA in its stable area.

Transmission-Line Background (Reflection and Transmission)

Voltage, current, or power emanating from a source impedance Z_S and delivered to a load Z_L can be regarded as the sum of incident and reflected waves traveling in opposite directions along a transmission line of characteristic impedance Z_o. If Z_L equals Z_o exactly, the incident wave is totally absorbed in the load and produces no reflected wave.

If Z_L differs from Z_o, some of the incident wave is not absorbed in the load, but is reflected back toward the source. If the source impedance Z_S equals Z_o, the reflected wave from the load is absorbed in the source and no further reflection occurs. For Z_S not equal to Z_o, a portion of the reflected wave from the load is re-reflected from the source back toward the load. For a lossless transmission line, this process repeats indefinitely.

The degree of mismatch between Z_o and Z_L (or Z_S) determines the amount of incident wave reflected. The ratio of reflected wave to incident wave is known as the reflection coefficient, and is simply a measure of the quality of the match between the transmission line and the terminating impedance. The reflection coefficient Γ is a complex quantity expressed in polar form as a magnitude and an angle:

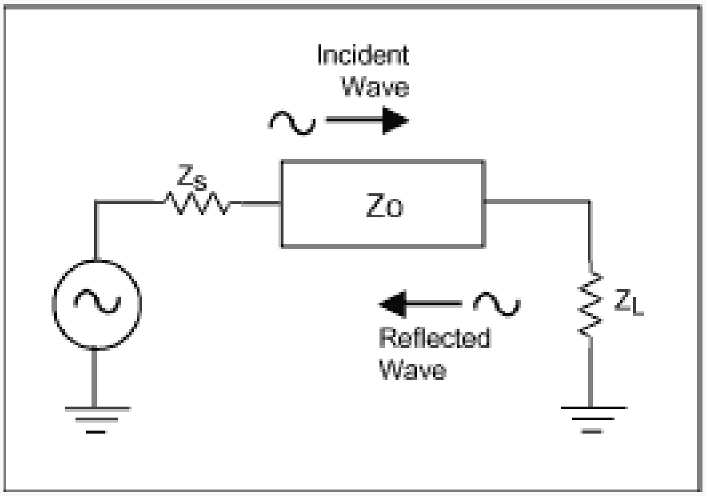

Figure 1.

$$\Gamma = \frac{\text{WAVEreflected}}{\text{WAVEincident}} = \rho/\theta \qquad \text{Eq. 1-1}$$

As matching between the characteristic impedance of the transmission line and the terminating impedance improves, the reflected wave becomes smaller. As a result, the reflection coefficient (Γ) decreases. When the match is perfect there is no reflected wave, and Γ equals zero. If the load impedance is an open or short circuit, no incident power can be absorbed in the load, and all must be reflected back toward the source. In that case, Γ is 1. The normal range of magnitude for Γ is between zero and one.

For reflection coefficients greater than one, the magnitude of the wave reflected from the load impedance should be greater than that of the incident wave to that load. It follows, therefore, that the load in question must be a source of power. This concept is useful in designing an oscillator, but at the input network of an amplifier it represents bad news. Reflection coefficients can be expressed in terms of the impedances under consideration. For example, the reflection coefficient at the load can be expressed as:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ or with } z_L = \frac{Z_L}{Z_0} \text{ in a normalized form } \Gamma = \frac{z_L - 1}{z_L + 1} \qquad \text{Eq. 1-2}$$

For impedance matching in microwave and RF networks, the source and load impedances are often expressed in terms of the source reflection coefficient (Γ_S) and load reflection coefficient (Γ_L). Incident and reflected waves are represented by wave flow graphs. Flow graphs let you build a graphic based on linear relations among different variables in the network, and they help in rapidly constructing a transfer function between two points in the network.

Each variable is represented by a node in the graph, and the different nodes are linked together by directive paths that give the relations between related and unrelated variables. Paths are directed from the node representing the independent variable to those representing the dependant variables, and to each path is assigned a gain related to the reflection coefficient linking the two variables.

Microwave load and power source:

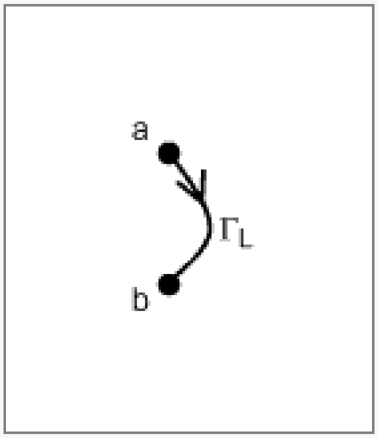

Figure 1-2a. Load flow graph.

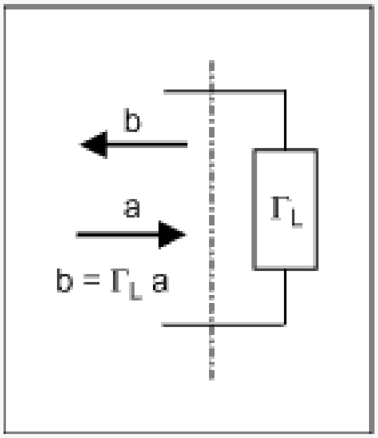

Figure 1-2b. Load reflection coefficient.

Variables a and b are complex values associated with the incident and reflected waves. The presence of variable b (reflected wave) depends on the presence of variable a (incident wave).

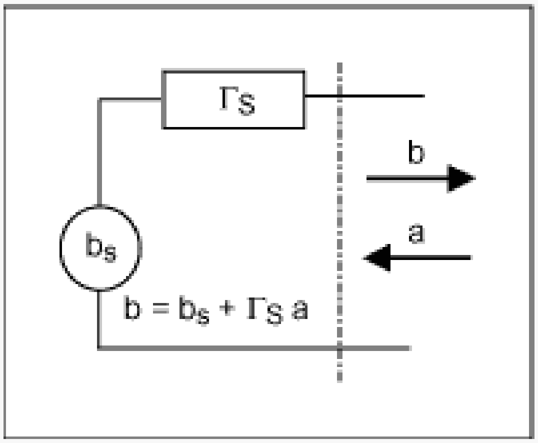

Figure 1-3a. Power source reflection coefficient.

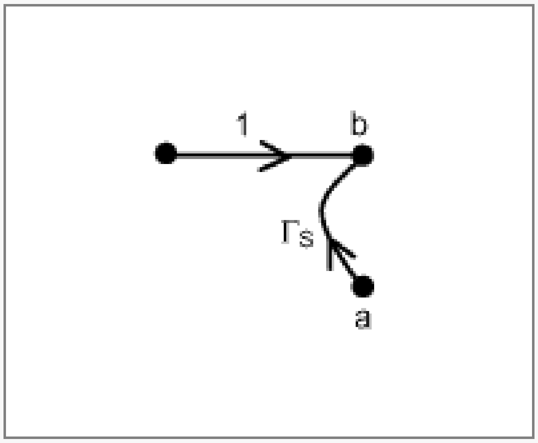

Figure 1-3b. Power source flow graph.

Γ_S = b/a is the reflection coefficient related to the power source when b_S = 0.

b_S = b is the incident wave from the power source when it supplies a 100% matched load.

S-Parameters and the Two-Port Network

By inserting a two-port network between source and load in the circuit of **Figure 1-3a**, we produce the circuit of **Figure 2-1**. The following may be said for any traveling wave that originates at the source:

- A portion of the wave that originates at the source and is incident on the two-port device (a1) will be reflected (b1), and another portion will be transmitted through the two-port device.
- A fraction of the transmitted signal is then reflected from the load and becomes incident on the output port of the two-port device (a2).
- A portion of the signal (a2) is then reflected from the output port back toward the load (b2), while a fraction is transmitted through the two-port device to the source.

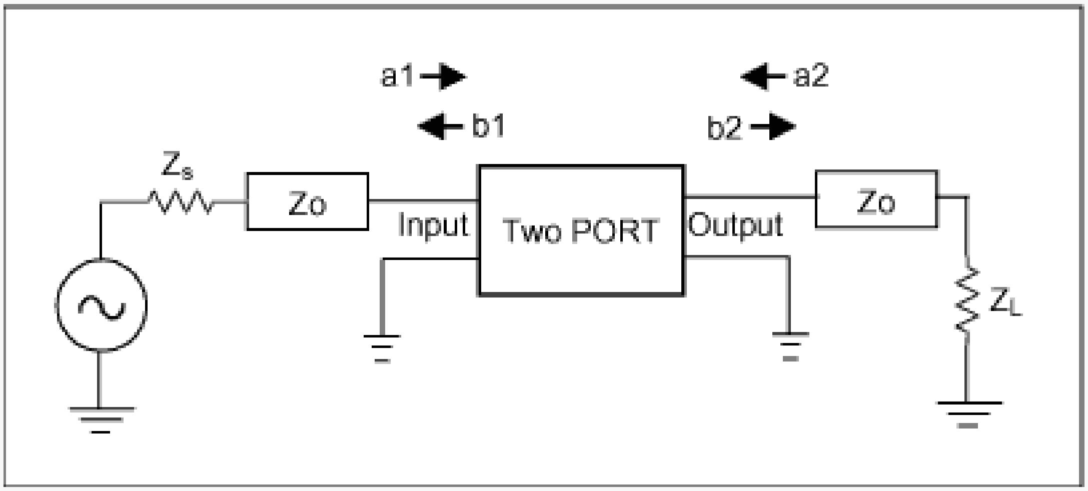

Figure 2-1.

It is obvious from the above discussion that any traveling wave present in the circuit is composed of two components. For instance, the traveling wave components flowing from the output of the two-port device to the load consist of the portion of a2 reflected from the output of the two-port device, and the portion of a1 transmitted through the two-port device. Similarly, the traveling wave flowing from the input of the two-port device back toward the source consists of the portion of a1 reflected from the input and the fraction of a2 transmitted through the two-port device. If we set these observations in equation form we get the following:

$b_1 = S_{11}a_1 + S_{12}a_2$

$b_2 = S_{21}a_1 + S_{22}a_2$.

Where:

S_{11} = the input reflection coefficient

S_{12} = the reverse transmission coefficient

S_{21} = the forward transmission coefficient

S_{22} = the output reflection coefficient with $\Delta S = (S_{11}S_{22} - S_{21}S_{12})$

If we set a_2 equal to zero, then $S_{11} = b_1/a_1 \mid a_2 = 0$.

If we set a_1 equal to zero, then $S_{22} = b_2/a_2 \mid a_1 = 0$.

By definition, a reflected wave divided by an incident wave is equal to the reflection coefficient, as noted above for the input and output of a two-port network. A two-port network (LNA) can be characterized completely by its scattering parameters (S-parameters). With these parameters, you can calculate potential instabilities (tendency to oscillate), maximum available gain, input and output impedances, and transducer gain. You can also calculate the optimum source and load impedances, either for simultaneous conjugate matching, or simply to help choose specific source and load impedances for a specified transducer gain.

Figure 2-2a shows a two-port network connection for available gain definition. Γin is the two-port input reflection coefficient, and the output is terminated in Γ_L. Γout is the two-port output reflection coefficient, as long as the input is terminated with Γ_S.

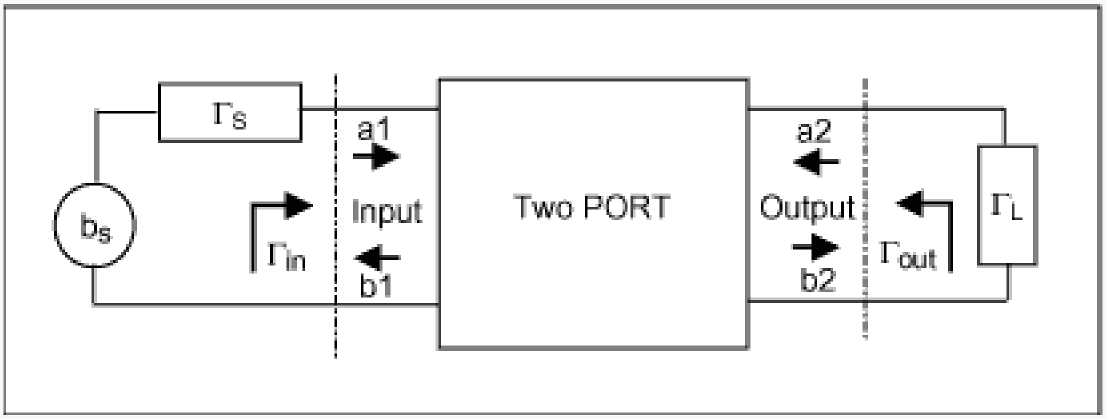

Figure 2-2a.

The flow graph for a loaded two-port network (**Figure 2-2b**) shows a closed loop that allows starting from one node and returning to that node without going through it twice. The precise physical significance of these loops is illustrated by our two-port network (an LNA for example), whose output is closed on a load Γ_L (**Figure 2-3**).

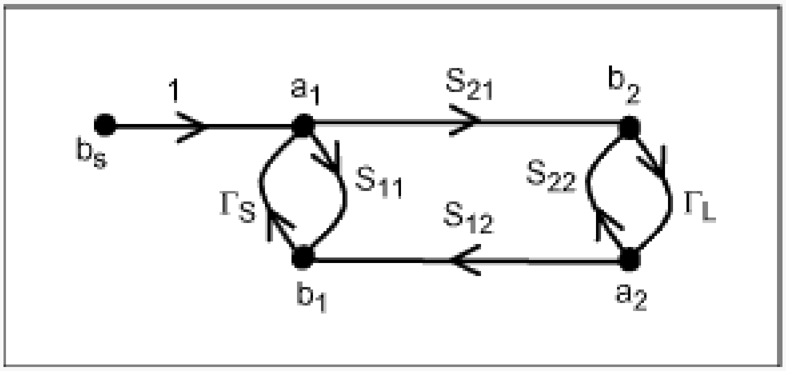

Figure 2-2b.

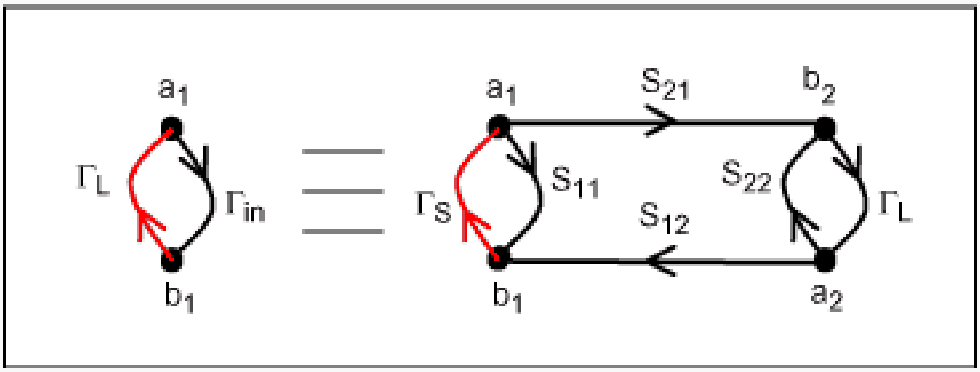

Figure 2-3.

To define the wave b_1 reflected back to the input, we just follow the different paths leading from a_1 to b_1. Possible paths are the following:

S_{11}, $S_{21}\Gamma_L S_{12}$, $S_{21}\Gamma_L S_{22}\Gamma_L S_{12}$, $S_{21}\Gamma_L S_{22}\Gamma_L S_{22}\Gamma_L S_{12}$, …

It appears that the loop $S_{22}\Gamma_2$ represents a multiple reflection due to the two-port network reflection coefficient between the output and the mismatched load.

$$b_1 = S_{11}a_1 + S_{21}\Gamma_L S_{12}a_1 \underbrace{\{1 + S_{22}\Gamma_L + \ldots\ldots + (S_{22}\Gamma_L)^n + \ldots\ldots\}}_{\dfrac{1}{1 - S_{22}\Gamma_L}}$$

The network input reflection coefficient for the two ports is:

$$\Gamma_{IN} = \frac{b_1}{a_1} = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L} \tag{Eq. 2-1}$$

When we close the two-port network input on a source impedance, the reflection coefficient seen from the output is (**Figure 2-4**):

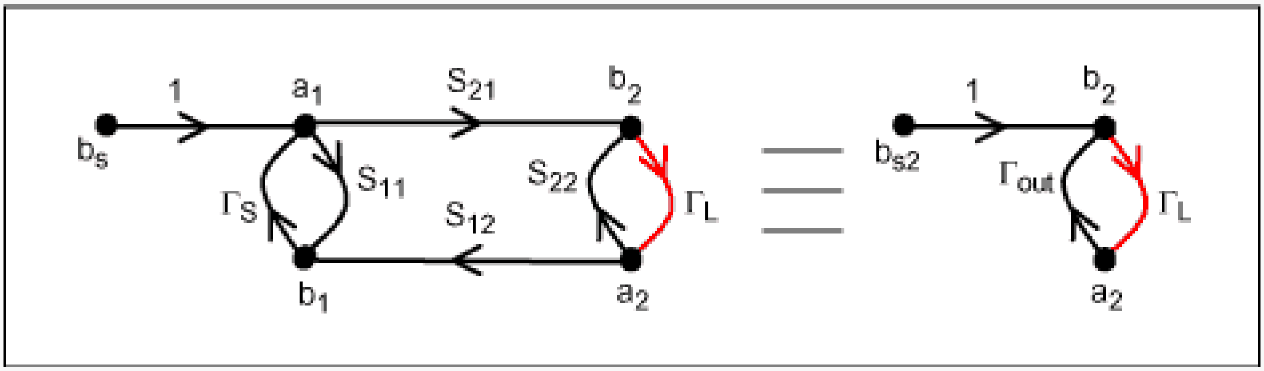

Figure 2-4.

$$\Gamma_{OUT} = \frac{b_2}{a_2} = S_{22} + \frac{S_{21}\Gamma_S S_{12}}{1 - S_{11}\Gamma_S} \tag{Eq. 2-2}$$

Gain for Two-Port Networks

Waves a and b are directly related to the concept of power. Unlike the low-frequency domain in which current and voltage gain are of primary interest, in the RF and microwave domains only power gain is considered. The different gains used by two-port devices are defined below, but first consider some facts regarding electrical power.

Maximum Available Power

The amount of power delivered to a load is easily determined, as shown by the connection of a source b_s, Γ_s to a load termination Γ_L (**Figure 3-1**). The power delivered by a source to a matched load is defined as the maximum available power (maximum power transferred) from the source when $\Gamma_s = \Gamma_L^*$. For these conditions, half the power is dissipated in the source and half is dissipated (transmitted) into the load. Figure 3-1 includes a flow graph.

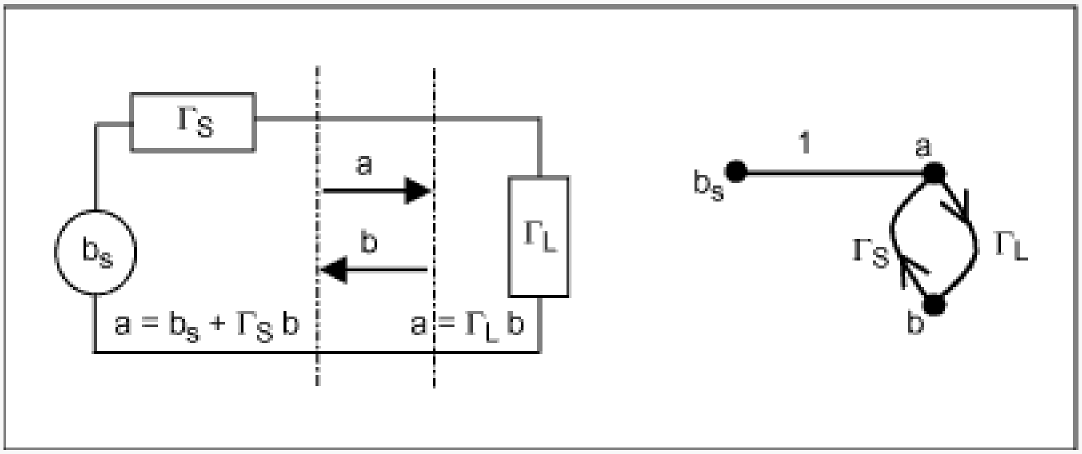

Figure 3-1.

The reflection seen from the source is:

$$a = b_S + b_S\Gamma_L\Gamma_S + \ldots\ldots + b_S(\Gamma_L\Gamma_S)^n \longrightarrow \frac{a}{b_S} = \frac{1}{1-\Gamma_L\Gamma_S} \qquad \text{Eq. 3-1}$$

Power transmitted to the load can be expressed as

$$P_2 = \frac{1}{2}(\,|a|^2 - |b|^2\,) = \frac{1}{2}|a|^2\,(1-|\Gamma_L|^2) \qquad \text{Eq. 3-2}$$

$$P_2 = \frac{1}{2}|b_S|^2\frac{1-|\Gamma_L|^2}{(1-\Gamma_L\Gamma_S)^2} \qquad \text{Eq. 3-3}$$

When $\Gamma_L = \Gamma_S^*$, the source can provide

$$P_{1AV} = \frac{|b_S|^2}{2(1-|\Gamma_L|^2)} \qquad \text{Eq. 3-4}$$

The asterisk indicates the complex conjugate, in which magnitude is the same but the angle has the opposite sign.

Two-Port Network Power Gain

Also defined as the operating gain or desired gain, it is the specific gain we want from a two-port network (LNA) for a particular application:

$$G = \frac{P_2}{P_1} \qquad \text{Eq. 3-5}$$

Where P_2 is power dissipated in the two-port output's load:

$$P_2 = \frac{1}{2}(|b_2|^2 - |a_2|^2) = \frac{1}{2}|b_2|^2\,(1-|\Gamma_L|^2) \qquad \text{Eq. 3-6}$$

Where P_1 is the power absorbed by the two-port input

$$P_1 = \frac{1}{2}(|b_1|^2 - |a_1|^2) = \frac{1}{2}|a_1|^2\,(1 - |\Gamma_{IN}|^2)$$

From the Figure 2-4 flow chart, we can easily extrapolate how b_2 is related to a_1:

$$b_S = S_{21}a_1\{1 + S_{22}\,\Gamma_L + \ldots\ldots + (S_{22}\Gamma_L)^n + \ldots\ldots\} \longrightarrow \frac{b_2}{a_1} = \frac{S_{21}}{1 - S_{22}\Gamma_L} \qquad \text{Eq. 3-7}$$

$$G = \frac{P_2}{P_1} = \frac{|b_2|^2(1 - |\Gamma_L|^2)}{|a_1|^2(1 - |\Gamma_{IN}|^2)} = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - S_{22}\Gamma_L)^2(1 - |\Gamma_{IN}|^2)} \qquad \text{Eq. 3-8}$$

With

$$\Gamma_{IN} = \frac{b_1}{a_1} = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L} \qquad \text{Eq. 2-1}$$

Two-port network power gain (operating or desired) is:

$$G = \frac{P_2}{P_1} = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - S_{22}\Gamma_L)^2\left(1 - \left|S_{11} + \frac{S_{21}\Gamma_L S_{12}}{(1 - S_{22}\Gamma_L)}\right|^2\right)} \qquad \text{Eq. 3-9}$$

$$G = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - S_{22}\Gamma_L)^2\,|S_{11} - \Delta_S\Gamma_L|^2}$$

Maximum Available Gain for Two-Port Network

The maximum gain available from a two-port network is a particular case of the transducer gain obtained when $\Gamma_L{}^* = \Gamma_{OUT}$. For a two-port network, the maximum available gain G_{AV} is defined as the ratio of power available at the output to power available from the source, mathematically expressed as $G_{AV} = P_{2AV}/P_{1AV}$. P_{2AV} is the available output power, and P_{1AV} is the power available from the source. Because maximum available gain can indicate whether an LNA has enough gain for the task, the calculation of this parameter is useful as a preliminary screening criteria for LNAs.

The initial wave b_{s2} seen from the two-port network output (Figure 2-4) is expressed as:

$$b_{S2} = \frac{b_S S_{21}}{(1 - S_{11}\Gamma_L)} \qquad \text{Eq. 4-1}$$

At the two-port input, the source is able to supply (Figure 2-2b and Figure 2-3):

$$P_{1AV} = \frac{1}{2}|b_S|^2\,\frac{(1 - |\Gamma_{IN}|^2)}{(1 - \Gamma_{IN}\Gamma_S)^2} \qquad \text{Eq. 4-2}$$

$$P_{1AV} = \frac{|b_S|^2}{2(1 - |\Gamma_S|^2)} \text{ When } \Gamma_{IN} = \Gamma_S{}^* \qquad \text{Eq. 4-3}$$

Power available at the two-port output is:

$$P_{2AV} = \frac{|b_{S2}|^2}{2(1 - |\Gamma_{OUT}|^2)} \quad \text{When } \Gamma_{OUT} = \Gamma_L^* \qquad \text{Eq. 4-4}$$

Expressed as a function of the two-port network's input source,

$$P_{2AV} = \frac{|b_S S_{21}|^2}{2(1 - |\Gamma_{OUT}|^2)(1 - S_{11}\Gamma_S)^2} \qquad \text{Eq. 4-5}$$

$$G_{AV} = \frac{P_{2AV}}{P_{1AV}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{(1 - |\Gamma_{OUT}|^2)(1 - S_{11}\Gamma_S)^2} \qquad \text{Eq. 4-6}$$

As shown by Eq. (4-7):

$\Gamma_{OUT} = S_{22} + (S_{21}\Gamma_S S_{12} / 1 - S_{11}\Gamma_S$

the maximum available gain factor depends on the source termination Γ_s and the two-port s-parameters. Maximum available gain depends on the two-port output being conjugately matched to Γ_L. Thus, $\Gamma_L = \Gamma_{out}^*$. As shown in Eq. 2-2, Γ_{out} is a function of Γ_S and the two-port parameters:

$$G_{AV} = \frac{P_{2AV}}{P_{1AV}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{(1 - |\Gamma_{OUT}|^2)(1 - S_{11}\Gamma_S)^2}$$

$$G_{AV} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{\left(1 - \left|S_{22} + \dfrac{S_{21}\Gamma_S S_{12}}{(1 - S_{11}\Gamma_S)}\right|^2\right)(1 - S_{11}\Gamma_S)^2} \qquad \text{Eq. 4-7}$$

The power available at the two-port output load is a function of power delivered to the two-port input, and power delivered to the two-port input depends on the mismatch between Γ_s and Γ_{in}. To determine power available from the source, you must terminate the source with a complex-conjugate load.

Transducer Gain

Transducer gain (the gain term most often referenced in RF-amplifier design) is defined as the output power P_2 delivered to a load by a source, divided by the maximum power available from the source. Transducer gain includes the effects of impedance matching at the input and output, as well as the contribution made by the LNA to the overall gain of the amplifier stage. Transducer gain neglects resistive losses in the components.

$$P_2 = 1/2|b_2|^2(1 - |\Gamma_L|^2) \quad \text{and } P_{1AV} = \frac{|bs|^2}{2(1 - |\Gamma_S|^2)} \quad \text{When } \Gamma_{IN} = \Gamma_S^*$$

$$G_T = \frac{P_2}{P_{1AV}} = \frac{|b_2|^2}{|b_S|^2}(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2) \qquad \text{Eq. 5-1}$$

To complete the transducer gain equation, define b_s (the source initial wave) as a function of b_{s2}, the initial wave seen from the two-port output (Figure 2-4), and b_2 as the wave fed back from the load Γ_L to the two-port network output.

$$b_2 = b_{S2} + b_{S2}\Gamma_{OUT}\Gamma_L + \ldots\ldots + b_{S2}(\Gamma_{OUT}\Gamma_L)^n \longrightarrow b_2 = \frac{b_{S2}}{1 - \Gamma_{OUT}\Gamma_L} \qquad \text{Eq. 5-2}$$

$$b_{S2} = \frac{b_S S_{21}}{(1 - S_{11}\Gamma_S)} \longrightarrow b_2 = \frac{b_S S_{21}}{(1 - S_{11}\Gamma_S)(1 - \Gamma_{OUT}\Gamma_L)} \qquad \text{Eq. 5-3}$$

As shown in Eq. 2-2, Γ_{OUT} is a function of Γ_S and the two-port parameters:

$$\Gamma_{OUT} = S_{22} + \frac{S_{21}\Gamma_S S_{12}}{1 - S_{11}\Gamma_S} \quad b_2 = \frac{b_S S_{21}}{(1 - S_{11}\Gamma_S)\left(1 - \left(S_{22} + \frac{S_{21}\Gamma_S S_{12}}{1 - S_{11}\Gamma_S}\right)\Gamma_L\right)} \qquad \text{Eq. 5-4}$$

Stability and maximum available gain (MAG) are two of the more important considerations in choosing a two-port network (LNA) for use in amplifier design. As used here, stability measures the tendency of an LNA to oscillate. Maximum available gain is a figure of merit for the LNA, which indicates the maximum theoretical power gain you can expect from the device when it is conjugately matched to its source and load impedances.

Transducer gain for a two-port network is:

$$G_T = \frac{P_2}{P_{1AV}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_S)(1 - \Gamma_{OUT}\Gamma_L)|^2}$$

$$G_T = \frac{P_2}{P_{1AV}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_S\Gamma_L|^2} \qquad \text{Eq. 5-5}$$

Reference

Bowick, Chris. *RF Circuit Designs*. Howard W. Sams & Co. Inc., a publishing subsidiary of ITT.

A similar version of this article appeared in the October 2001 issue of *Microwaves and RF* magazine.

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Jan 09, 2003

APPLICATION NOTE 1851

Low-Noise Amplifier Stability Concept to Practical Considerations, Part 2

Abstract: Part two of a three-part series. This part covers the RF aspects of low-noise amplifiers. Stability, impedance matching and general amplifier design are covered. Emphasis is on S-parameters as design tools.

- Part 1
- Part 2
- Part 3

In Part 1, we started our discussion with a brief background on transmission lines and a reminder about RF power gain definitions. In this part, we jump into the RF aspect of low noise amplifiers by examining stability (tendency for oscillation), impedance matching, and general amplifier design, using scattering parameters (S-parameters) as design tools.

Part 3 completes the series by presenting application examples. The first shows how to match an LNA in the maximum available gain condition. The second deals with an LNA matched in the constant desired gain condition. The third exercise stresses the importance of matching a potentially unstable LNA in its stable area.

Stability Calculations

Some say the easiest way to build an oscillator is to design an amplifier. Experience shows this to be true, but it need not be the case. A bit of planning and some basic, a priori knowledge of the LNA to be used can go a long way toward preventing oscillation in an amplifier design.

Two-port networks can be completely characterized by their scattering (S) parameters. Scattering parameters allow the calculation of potential instabilities (trend toward oscillation), maximum available gain, input and output impedances, and transducer gain. S-parameters also allow the calculation of optimum source and load impedances, either for simultaneous conjugate matching or simply to help choose the source and load impedances for a specified transducer gain.

One characteristic of two-port networks is that $|S_{21}| \neq |S_{12}|$, and that most of the time $|S_{21}| >> |S_{12}|$. Consider the following network (an LNA), connected to its source and load impedances. The magnitudes $|\Gamma_S|$ and $|\Gamma_L|$ of the source and load reflection coefficients Γ_S and Γ_L are less than or equal to 1, which means that the corresponding impedances have positive real parts.

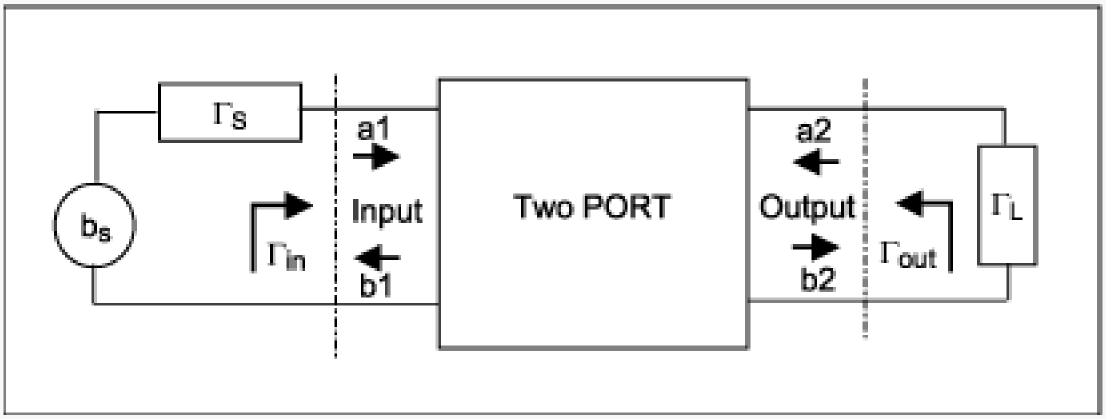


Figure 6-1.

First, we study a one-sided two-port network for which $S_{12} = 0$. This case is interesting, because it supports a simple example that helps to highlight certain rules without the need for heavy mathematics.

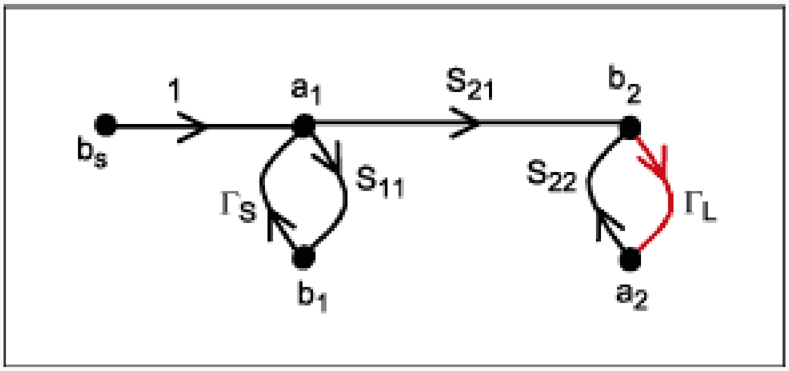


Figure 6-2.

In this case we write a simplified version of the transducer gain equation (Eq. 5-5) as:

$$G_T = \underbrace{\frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}}_{G_1} |S_{21}|^2 \underbrace{\frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}}_{G_2} \qquad \text{Eq. 6-1}$$

G_1 and G_2 are called the mismatch gain.

When $|S_{22}| < 1$ and $|S_{11}| < 1$, the two-port network is unconditionally stable. As a consequence, the amplifier remains stable for any combination of source and load impedances. Without reverse gain ($S_{12} = 0$), the maximum available gain you can hope to achieve with an LNA under the conjugately matched condition ($\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$) is:

$$G_{TMAX} = \frac{1}{|1 - S_{11}|^2} |S_{21}|^2 \frac{1}{|1 - S_{22}|^2} \qquad \text{Eq. 6-2}$$

These conjugate-matching conditions assume the maximum power transfer from source to load. With no reverse gain ($S_{12} = 0$), the source and load impedance have no effect on the device input and output impedance.

When $|S_{22}| > 1$ or $|S_{11}| > 1$, the two-port network is potentially unstable, and will oscillate for certain values of source and load impedance $\Gamma_S = 1/S_{11}$ or $\Gamma_L = 1/S_{22}$. The conditions $|S_{22}| > 1$ or $|S_{11}| > 1$ do not indicate, however, that the network cannot be used as an amplifier. It merely indicates that oscillations may occur if you fail to exercise extreme care in choosing the source and load impedances.

Two-Port Network Stability in the General Case (S_{12} Different from Zero)

A two-port network is unconditionally stable if the reflection coefficients Γ_{in} and Γ_{out} (seen from input and output) have magnitudes < 1, regardless of the loads $\Gamma_S \leq 1$ and $\Gamma_L \leq 1$ at the two-port network input and output access. Impedances seen from the input and output have positive real parts, and are independent of the load impedances presented at the input and output access.

$\Gamma_{IN} = S_{11} + (S_{21}\Gamma_L S_{12} / 1 - S_{22}\Gamma_L)$ can also be written as $\Gamma_{IN} = (S_{11\ -\ \Delta S}\Gamma_L / 1 - S_{22}\Gamma_L)$ and transformed as

$$\Gamma_{IN} = \frac{\Delta S}{S_{22}} + \frac{S_{21}S_{12}}{S_{22}} \frac{1}{1 - S_{22}\Gamma_L} \qquad \text{Eq. 7-1}$$

with $\Delta S = (S_{11}S_{22} - S_{21}S_{12})$.

Reflection coefficient Γ_L is a complex value with magnitude $|\Gamma_L| \leq 1$. Consequently, it will be located inside a circle C_1, centered at the origin (O_1) with radius = 1. The location of Γ_{in} can be determined by a succession of simple geometrical transformations including multiplication ratios, shiftings, and inversions:

$$S_{22}\Gamma_L \quad \begin{array}{l} \longrightarrow \text{multiplied by a } S_{22} \text{ ratio} \\ \longrightarrow \text{Circle C2: } O_2 = 0 \ ; \ R_2 = |S_{22}| \end{array} \qquad \text{Eq. 7-2}$$

$$1 - S_{22}\Gamma_L \quad \begin{array}{l} \longrightarrow \text{Right shifting from C2 by a } +1 \\ \longrightarrow \text{Circle C3: } O_3 = 1 \ ; \ R_3 = |S_{22}| \end{array} \qquad \text{Eq. 7-3}$$

$$\frac{1}{1 - S_{22}\Gamma_L} \quad \begin{array}{l} \longrightarrow \text{Inversion in the complex map} \\ \longrightarrow \text{Circle C4: } O_4 = \dfrac{1}{1 - |S_{22}|^2} \ ; \ R_4 = \dfrac{|S_{22}|}{|1 - |S_{22}|^2|} \end{array} \qquad \text{Eq. 7-4}$$

$$\frac{S_{21}S_{12}}{S_{22}} \frac{1}{1 - S_{22}\Gamma_L} \quad \begin{array}{l} \longrightarrow \text{multiplied by a } S_{21}\ S_{12}/S_{22} \text{ ratio} \\ \longrightarrow \text{Circle C5: } O_5 = \dfrac{S_{21}S_{12}}{S_{22}} \dfrac{1}{1 - |S_{22}|^2} \ ; R_5 = \dfrac{|S_{21}S_{12}|}{|1 - |S_{22}|^2|} \end{array} \qquad \text{Eq. 7-5}$$

$$\frac{\Delta S}{S_{22}} + \frac{S_{21}S_{12}}{S_{22}} \frac{1}{1 - S_{22}\Gamma_L} \quad \begin{array}{l} \longrightarrow \text{Shifting from C2 by } \Delta S/S_{22} \\ \longrightarrow \text{Circle C6: } O_6 = \dfrac{\Delta S}{S_{22}} + \dfrac{S_{21}S_{12}}{S_{22}(1 - |S_{22}|^2)} \ ; R_6 = \dfrac{|S_{21}S_{12}|}{|1 - |S_{22}|^2|} \end{array} \qquad \text{Eq. 7-6}$$

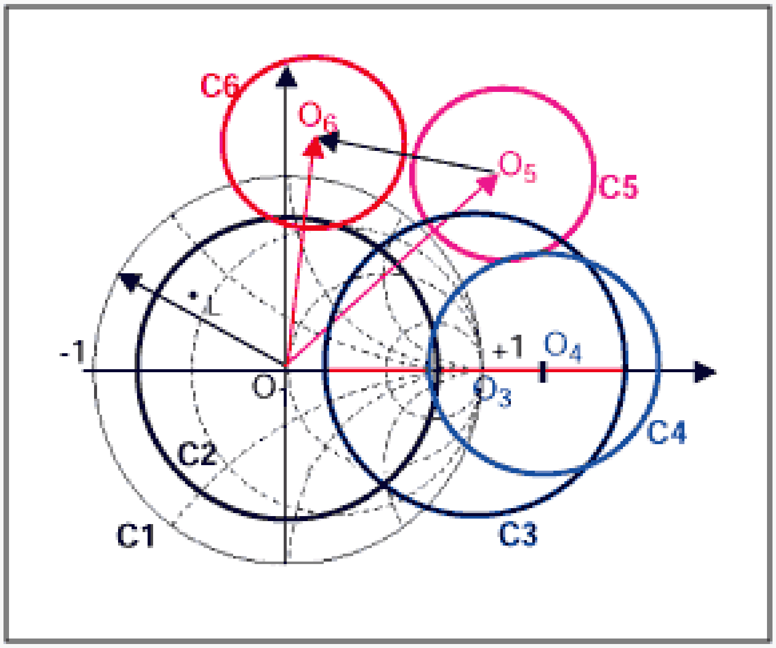


Figure 7-1.

When the two-port network output reflection coefficient $S_{22} < 1$, the radius of circle C4 is too small (with respect to its center location O_4) to surround the center of C1. From the relation between Γ_{in} and Γ_L, it appears that if Γ_L is included inside the circle C1, then Γ_{in} is included inside the circle C6 when $S_{22} < 1$. The magnitude of the reflection coefficient Γ_{in} (radius R6) seen from the two-port network's input is inside the circle C6. Therefore, $\Gamma_{in} \le 1$ if $R_6 = |S_{21}S_{12}| / |1 - |S_{22}|^2| \le 1$, and then $|S_{21}S_{12}| \le 1 - |S_{22}|^2$.

Considering that the radius of circle C1 is 1, we can tell that the two-port network will be unconditionally stable if C6 is totally included inside the circle C1. As a consequence, $|O_6| + R_6 \le 1$.

$$|O_6|^2 \le 1 - |R_6|^2 \qquad \left(\frac{\Delta S}{S_{22}} + \frac{S_{21}S_{12}}{S_{22}} \frac{1}{1 - |S_{22}|^2}\right)^2 \le \left(1 - \frac{|S_{21}S_{12}|}{|1 - |S_{22}|^2|}\right)^2 \qquad \text{Eq. 7-7}$$

O_6 and R_6 are complex values. To obtain their magnitudes we must square the complex values:

$|O_6|^2 = O_6 \times O_6^*$.

This last transformation gives the stability factor K, called the Linvill stability factor:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2}{2|S_{21}S_{12}|} \ge 1 \qquad \text{Eq. 7-8}$$

Once the condition of unconditional stability has been verified for the reflection coefficient Γ_{in} seen from the two-port network input, a similar verification can be made for the output reflection coefficient $\Gamma_{OUT} = S_{22} + (S_{21}\Gamma_s S_{12} / 1 - S_{11}\Gamma_s)$ seen from the two port network output. This leads to the results $|S_{21}S_{12}| \le 1 - |S_{11}|^2$ and thus Eq. 7-8.

Thus, we have verified that a two port network is unconditionally stable when:

Input : $|S_{21}S_{12}| \le 1 - |S_{11}|^2$

Input : $|S_{21}S_{12}| \le 1 - |S_{11}|^2$

And the Linvill stability factor (Eq. 7-8) are verified.

Combining the stability conditions on input and output yields the following for a two-port network:

The overall stability condition

$$|\Delta S| = |S_{11}S_{22} - S_{21}S_{12}| \le 1 \qquad \text{Eq. 7-9}$$

and

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2}{2|S_{21}S_{12}|} \ge 1$$

When K is greater than 1 and ΔS is less than one, the device is unconditionally stable for any combination of source and load impedances. If, on the other hand, K calculates to be less than 1 or ΔS to be greater than 1, the device is potentially unstable and is likely to oscillate for certain combinations of source and load impedance. With K less than 1, or ΔS greater than 1, you must be extremely careful in choosing the source and load impedances. Those conditions do not mean that the device cannot be used, but merely that it will be more difficult to use.

Potentially Unstable Condition

When the Linvill stability factor indicates potential instability, the device is likely to oscillate with some combination of source and load impedance. When K calculates to be less than 1, therefore, or $|\Delta S|$ to be more than 1, it is important to choose the source and load impedances very carefully. One of the best methods for determining which source and load impedances will cause the device to be unstable is to plot stability circles on a Smith chart.

A stability circle is simply a circle on the Smith chart that represents the boundary between those values of source and load impedance that cause instability and those that do not. The perimeter of the circle is the locus of points that force K = 1. Either the inside or the outside of the circle can represent the unstable region, and that determination is made after the circle is drawn. The centers and radii of the input and output stability circles are found as follows.

When a two-port network (LNA) is potentially unstable, we need to identify its stable areas (the values of Γ_S and Γ_L for which $|\Gamma_{in}| \le 1$ and $|\Gamma_{out}| \le 1$). To define the boundary between stability and instability, we consider the maximum possible value (unity) of the reflection coefficients Γ_{in} and Γ_{out} seen from the device input and output:

$$\Gamma_{IN} = \frac{S_{11} - \Delta S \Gamma_L}{1 - S_{22}\Gamma_L} = 1 \qquad \text{Eq. 8-1}$$

$$\Gamma_{OUT} = \frac{S_{22} - \Delta S \Gamma_S}{1 - S_{11}\Gamma_S} = 1 \qquad \text{Eq. 8-2}$$

For the device output, we demonstrate that the output stability circle C2 (with center O_2 and radius R_2) is defined by the Γ_L values for which $|\Gamma_{in}| = 1$:

$$|S_{11} - \Delta S\Gamma_L|^2 = |1 - S_{22}\Gamma_L|^2$$
$$|S_{11}|^2 + |\Delta S\Gamma_L|^2 - 2\mathrm{Real}(S_{11}{}^{*}\Delta S\Gamma_L) = 1 + |S_{22}\Gamma_L|^2 - 2\mathrm{Real}(S_{22}\Gamma_L) \qquad \text{Eq. 8-3}$$

$$|\Gamma_L|^2 - 2\mathrm{Real}(A\Gamma_L) + B = 0, \text{ with A and B defined as:} \qquad \text{Eq. 8-4}$$

$$A = \frac{S_{22} - S_{11} * \Delta S}{|S_{22}|^2 - |\Delta S|^2} \qquad \text{Eq. 8-6}$$

$$B = \frac{1 - |S_{11}|^2}{|S_{22}|^2 + |\Delta S|^2} \qquad \text{Eq. 8-7}$$

$$|\Gamma_L|^2 - 2\mathrm{Real}(A\Gamma_L) + A^2 - A^2 + B = 0 \qquad \text{Eq. 8-8}$$

The above leads to the equation of a circle in the complex plane: $|\Gamma_L - A^*|^2 = |A|^2 - B$, centered at A^* with a radius of $(|A|^2 - B)^{1/2}$:

$$O_2 = A^* = \frac{S_{22}{}^{*} - S_{11}\Delta S^*}{|S_{22}|^2 - |\Delta S|^2} \qquad \text{Eq. 8-9}$$

$$R_2 = (|A|^2 - B)^{1/2} = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta S|^2\right|} \qquad \text{Eq. 8-10}$$

The same demonstration can be made for the device input, with Γ_S and $|\Gamma_{out}|$:

Stability circles for device:

$$\begin{aligned}&\text{Input:} \quad O_1 = (S_{11}{}^{*} - S_{22}\Delta S^*)/(|S_{11}|^2 - |\Delta S|^2) \quad R1 = (|S_{12}S_{21}|)/\left||S_{11}|^2 - |\Delta S|^2\right| \\ &\text{Output:} \quad O_2 = (S_{22}{}^{*} - S_{11}\Delta S^*)/(|S_{22}|^2 - |\Delta S|^2) \quad R2 = (|S_{12}S_{21}|)/\left||S_{22}|^2 - |\Delta S|^2\right|\end{aligned} \qquad \text{Eq. 8-11}$$

To determine whether circle C_2 surrounds the Smith chart origin, we calculate $|O_2|^2 - R_2{}^2$. If $|O_2| > R_2$, the circle does not surround the origin (**Figure 8-1**). If $|O_2| < R_2$, it does:

$$|O_2|^2 - R^2 = \frac{1 - |S_{11}|^2}{|S_{22}|^2 - |\Delta S|^2} \qquad \text{Eq. 8-12}$$

If $(1 - |S_{11}|^2)$ and $(|S_{22}|^2 - |\Delta S|^2)$ have the same sign (positive or negative), then $|O_2|$ is greater than R_2, and circle C_2 does not surround the Smith chart origin. If $(1 - |S_{11}|^2)$ and $(|S_{22}|^2 - |\Delta S|^2)$ have different signs, $|O_2|$ is less than R_2, and circle C_2 does surround the Smith chart origin. After making the stability calculations, you can plot stability circles directly on the Smith chart.

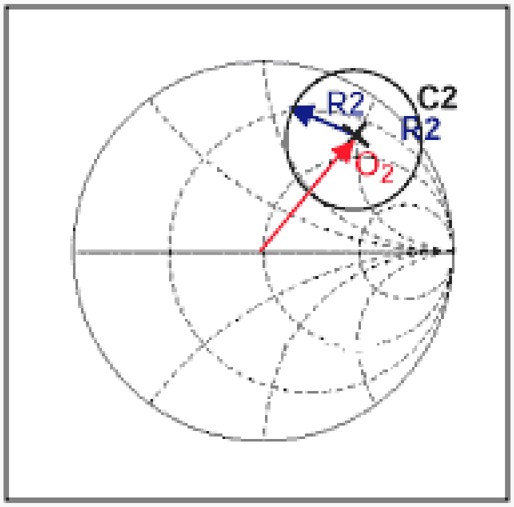
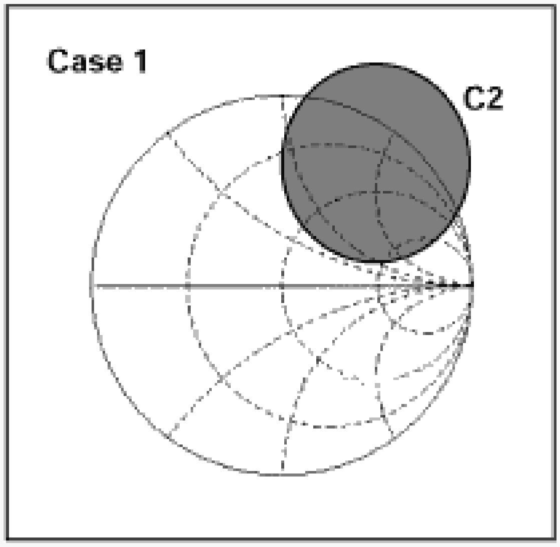


Figure 8-1.

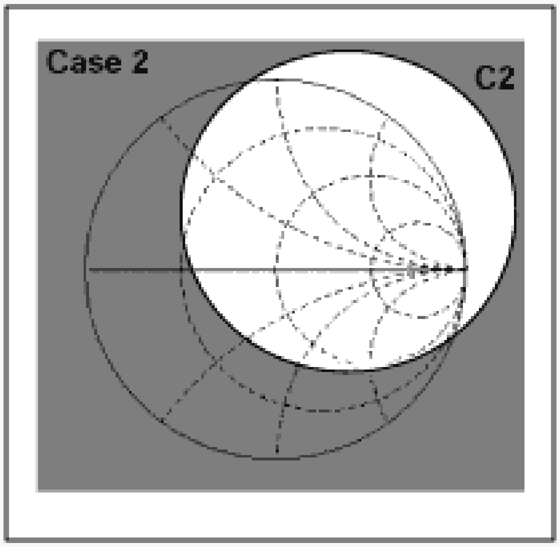
For a potentially unstable amplifier, the stability circles might appear as in **Figure 8-2 (a-d)**. As shown, the chart is often intersected by only a portion of the stability circle. After plotting stability circles on the chart, the next step is to determine which side of the circle (inside or outside) represents the stable region.



Output Stability circle $|S_{11}| < 1$ and $|\Delta S| < |S_{22}|$

Input Stability circle $|S_{22}| < 1$ and $|\Delta S| < |S_{11}|$

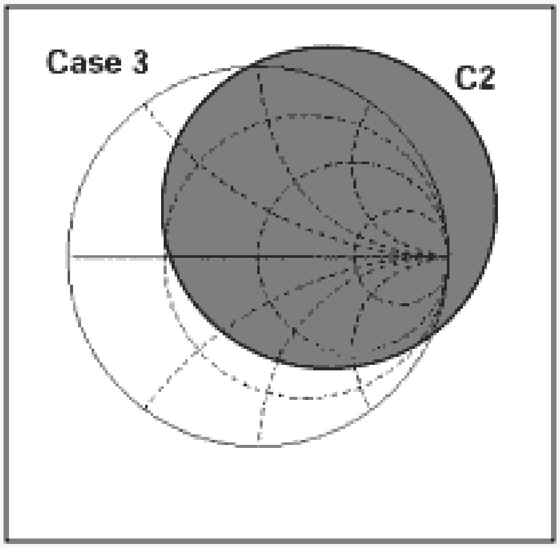
Figure 8-2a. Circle C_2 doesn't surround the Smith Chart origin.



Output Stability circle $|S_{11}| < 1$ and $|\Delta S| > |S_{22}|$

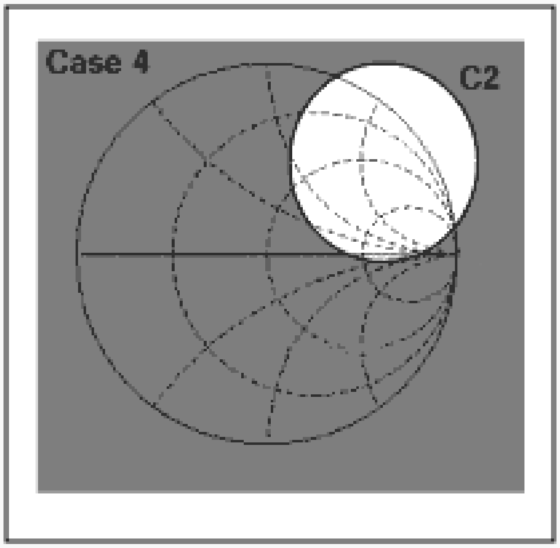
Input Stability circle $|S_{22}| < 1$ and $|\Delta S| > |S_{11}|$

Figure 8-2b. Circle C_2 surrounds the Smith Chart origin.



Output Stability circle $|S_{11}| > 1$ and $|\Delta S| < |S_{22}|$
Input Stability circle $|S_{22}| > 1$ and $|\Delta S| < |S_{11}|$

Figure 8-2c. Circle C_2 surrounds the Smith Chart origin.



Output Stability circle $|S_{11}| > 1$ and $|\Delta S| > |S_{22}|$
Input Stability circle $|S_{22}| > 1$ and $|\Delta S| > |S_{11}|$

Figure 8-2d. Circle C_2 doesn't surround the Smith Chart origin.

Grey areas on the figures represent regions of instability. Choosing Γ_L such that $\Gamma_{IN} < 1$ ensures that the two-port network is stable at the output. The output conditions in terms of Γ_S and Γ_{IN} are also true for the input in terms of Γ_L and Γ_{OUT}. To design an oscillator, for example, we might choose Γ_L such that $\Gamma_{IN} > 1$. That is easily done if S_{11} and S_{22} for the device are less than 1.

Because the S parameters were measured with a 50Ω source and load, and because the LNA remained stable for those conditions (S_{11} and $S_{22} < 1$), the center of the normalized Smith chart must be part of the stable region as described by the stability circles. If, therefore, one of the stability circles surrounds the center of the chart in this case, the inside of that circle represents the region of stable impedances for that port.

On the other hand, if S_{11} or $S_{22} > 1$ for an unstable LNA, the circle does not surround the center of the chart. Therefore, the entire area outside that circle represents the stable operating region for the port. Because it is generally wise to do front-end design with the LNA intrinsically stable ($S_{11} < 1$ and $S_{22} < 1$), only cases 1 and 2 are generally encountered in practical applications. For unconditionally stable amplifiers the entire Smith chart represents a stable operating region (**Figure 8-3**). Thus, you may never find stability circles to plot for an unconditionally stable LNA.

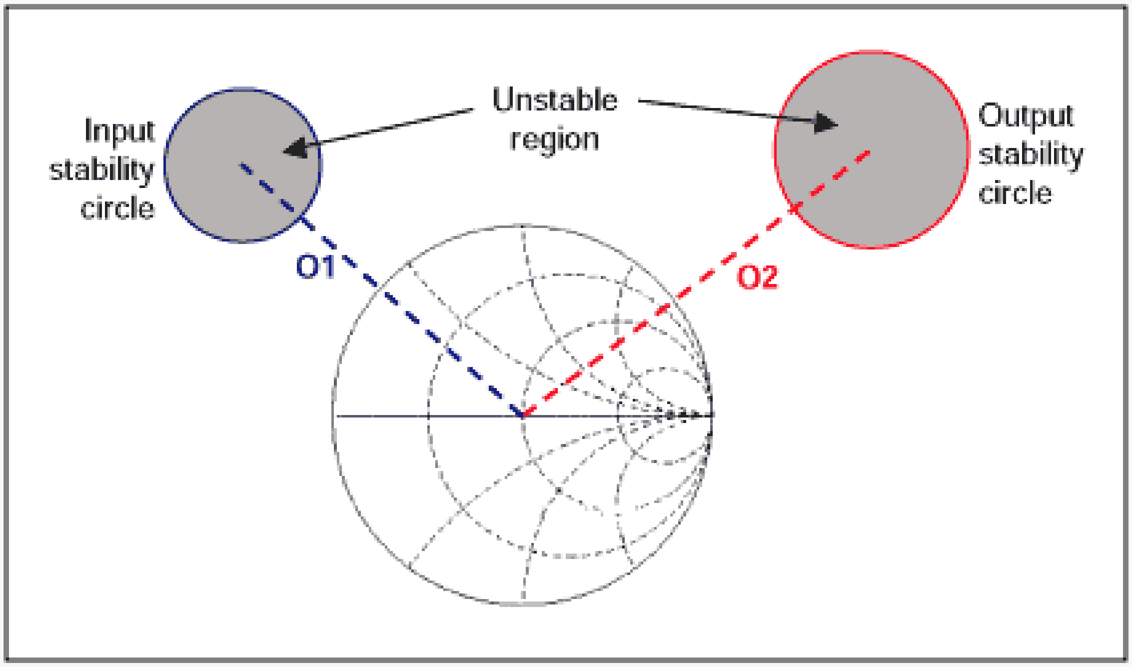


Figure 8-3.

Simultaneous Conjugate Matching (Unconditionally Stable LNA)

For simultaneous conjugate matching of an LNA, it must operate at its maximum available gain. Maximum available gain is the most gain you can expect from a two-port network under the conjugately matched condition.

You can proceed with the design when you find an LNA whose gain capability matches your requirements. The following design procedure yields source and load reflection coefficients that provide a conjugate match for the LNA's actual input and output impedances. Remember that the output impedance of an LNA depends on the source impedance "seen" by the LNA. Conversely, the input impedance of the LNA depends on the load impedance seen by the LNA. These dependencies are caused by the LNA's reverse gain (S_{12}). If S_{12} is equal to zero, the load and source impedances have no effect on the LNA's input and output impedances.

We now demonstrate how to find the desired load reflection coefficient for a conjugate match, and also how to express the two-port network transducer gain $G_T = P_2 / P_{1AV}$ as a function of the stability coefficient K. Transducer gain is maximum when power entering the two-port network is maximum ($P_1 = P_{1AV}$) and when power dissipated in the load is maximum ($P_2 = P_{2AV}$). That is, when $\Gamma_S = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$:

$$\Gamma_S = \frac{S_{11}^* - \Delta S^* \Gamma_L^*}{1 - S_{22}^* \Gamma_L^*} = \Gamma_{IN}^* \tag{Eq. 9-1}$$

$$\Gamma_L = \frac{S_{22}^* - \Delta S^* \Gamma_S^*}{1 - S_{11}^* \Gamma_S^*} = \Gamma_{OUT}^* \tag{Eq. 9-2}$$

By replacing Γ_L with its value in the equation for Γ_S, we can perform the calculations with the input reflection coefficient Γ_S:

$$\Gamma_S = \frac{S_{11}^* - \Delta S^* \left(\frac{S_{22} - \Delta S \Gamma_S}{1 - S_{11} \Gamma_S} \right)}{1 - S_{22}^* \left(\frac{S_{22} - \Delta S \Gamma_S}{1 - S_{11} \Gamma_S} \right)} \tag{Eq. 9-3}$$

Next, substitute the calculated value for Γ_S (above) in the 2nd-order equation below:

$$\underbrace{(S_{11} - S_{22}{}^{*}\Delta S)}_{A_1 = C_1} \Gamma_S{}^2 - \underbrace{(1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta S|^2)}_{B_1} \Gamma_S + \underbrace{S_{11}{}^{*} - S_{22}\Delta S^{*}}_{C_1{}^{*}} = 0 \qquad \text{Eq. 9-4}$$

Thus,

$$\Gamma_S = \frac{B_1 \pm \sqrt{(B_1{}^2 - 4|C_1|^2)}}{2C_1} \qquad \text{Eq. 9-5}$$

B_1 and C_1 have to be calculated first, because their values determine which sign (plus or minus) must appear before the radical above. Depending on whether the sign is positive or negative, two solutions Γ_S" and Γ_S"" are possible for Γ_S:

$$\Gamma_S{}'' \, \Gamma_S{}'''' = \frac{C_1{}^{*}}{C_1} \longrightarrow |\Gamma_S{}''| \, |\Gamma_S{}''''| = 1 \qquad \text{Eq. 9-6}$$

Of the two solutions Γ_S" or Γ_S"", only the one with magnitude < 1 is valid. Making the same calculation with output reflection coefficient Γ_L gives a similar result:

$$\Gamma_L = \frac{B_2 \pm \sqrt{(B_2{}^2 - 4|C_2|^2)}}{2C_2} \qquad \text{Eq. 9-7}$$

With $C_2 = S_{22} - S_{11}{}^{*}\Delta S$, and $B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta S|^2$.

Next, calculate $(B_1{}^2 - 4|C_1|^2)$ and $(B_2{}^2 - 4|C_2|^2)$ to determine the input and output reflection coefficients (Γ_S and Γ_L) as a function of the stability factor K. Calculating the input as an example,

$$\begin{aligned} &|C_1|^2 = |S_{11} - S_{22}\,\Delta S^{*}|^2 = |S_{11}|^2 + |S_{22}\Delta S|^2 - 2\mathrm{Real}(S_{11}S_{22}{}^{*}\Delta S), \\ &\text{with } \mathrm{Real}(S_{11}S_{22}{}^{*}\Delta S) = |S_{11}S_{22}|^2 - \mathrm{Real}(S_{11}{}^{*}S_{22}{}^{*}\,S_{12}S_{21}), \text{ and} \\ &|\Delta S|^2 = |S_{11}S_{22}|^2 + |S_{12}S_{21}|^2 - 2\mathrm{Real}(S_{11}{}^{*}S_{22}{}^{*}\,S_{12}S_{21}). \end{aligned} \qquad \text{Eq. 9-8}$$

By replacing the expression $2\mathrm{Real}(S_{11}S_{22}{}^{*}\Delta S)$ in the ΔS function we can write:

$$\begin{aligned} &|C_1|^2 = (|S_{11}|^2 - |\Delta S|^2)\,(|1 - S_{22}|^2) + |S_{12}S_{21}|^2 \\ &B_1{}^2 = (1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta S|^2)^2 \text{ becomes } B1^2 = (1 - |S_{22}|^2)^2 + \\ &\qquad (|S_{11}|^2 - |\Delta S|^2)^2 + 2(1 - |S_{22}|^2)\,(|S_{11}|^2 - |\Delta S|^2). \end{aligned} \qquad \text{Eq. 9-9}$$

Finally,

$$B_1{}^2 - 4|C_1|^2 = (1 - |S_{22}|^2 - |S_{11}|^2 + |\Delta S|^2)^2 - 4|S_{12}S_{21}|^2 \qquad \text{Eq. 9-10}$$

can be expressed as a function of the stability factor K:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2}{2|S_{21}S_{12}|} \qquad \text{Eq. 9-11}$$

As a result, $B_1^2 - 4|C_1|^2 = 4|S_{12}S_{21}|^2(K^2 - 1)$.

We can now write Γ_S (and Γ_L, by changing the index from 1) to 2) in terms of the stability factor K:

For the input:

$$\Gamma_S = \frac{B_1 \pm 2|S_{12}S_{21}|\sqrt{(K^2 - 1)}}{2C_1} \qquad \text{Eq. 9-12}$$

For the output:

$$\Gamma_L = \frac{B_2 \pm 2|S_{12}S_{21}|\sqrt{(K^2 - 1)}}{2C_2} \qquad \text{Eq. 9-13}$$

To define the sign of Γ_S and Γ_L, we can use Γ_S for the calculations. If the stability factor K > 1, $B_1^2 - 4|C_1|^2 > 0$ in this case, and the input reflection coefficient's magnitude (value squared) is:

$$|\Gamma_S|^2 = \frac{B_1^2 + (B_1^2 - 4|C_1|^2) + 2(\pm)B_1\sqrt{(B_1^2 - 4|C_1|^2)}}{4|C_1|^2} \qquad \text{Eq. 9-14}$$

We now determine which sign before the radical (plus or minus) leads to the required condition $|\Gamma_S| < 1$:

$$\left[2(B_1^2 - 4|C_1|^2) + 2(\pm)B_1\sqrt{(B_1^2 - 4|C_1|^2)}\right] < 0 \qquad \text{Eq. 9-15}$$

$$2\sqrt{(B_1^2 - 4|C_1|^2)}\left[\sqrt{(B_1^2 - 4|C_1|^2)} + (\pm)B_1\right] < 0$$

$$\text{as a result,} \sqrt{(B_1^2 - 4|C_1|^2)} < -(\pm)B_1$$

As the stability factor K > 1 represents the unconditionally stable condition, we have demonstrated above that:

B1 > 0 → the - sign must be used in Γ_S equation for $\Gamma_S < 1$

B1 < 0 → the + sign must be used in Γ_S equation for $\Gamma_S < 1$

The sign preceding the radical is the opposite of the sign of B_1 (previously calculated in Eq. 9-4). The angle of the source reflection coefficient is simply the negative (conjugate) of the angle of the C_1 parameter (also found in Eq. 9-4). These relations are also true for calculating the output-matching conditions with $B_2 > 0$ and Γ_L.

When the desired source reflection coefficient is found, it can be plotted on a Smith chart, and the corresponding source impedance can be found directly. Or if you prefer, you can substitute Γ_S into Eq. 1-2, and solve for Z_S mathematically. With the desired source coefficient specified, you can calculate the load-reflection coefficient needed to properly terminate the LNA output (Eq. 9-2):

$\Gamma_L = [S_{22} + (S_{21}\Gamma_S S_{12} / 1 - S_{11}\Gamma_S)]^*$

Again, the asterisk indicates that you should take the conjugate of the quantity in parentheses (i.e., same magnitude but opposite sign for the angle). In other words, the magnitude of the result within the parentheses will be correct but the angle will have the wrong sign. Simply change the sign of the angle. Once the desired load reflection coefficient is found, it can be plotted on a Smith chart or substituted into Eq. 1-2 to find the corresponding load impedance.

Next, we verify the conjugate matching condition with stability coefficient K < 1 (the condition indicating a potentially unstable LNA). In that case $B_1^2 - 4|C_1|^2 < 0$ (Eq. 9-11). To obtain the roots of this expression, it should be written as an imaginary value: $B_1^2 - 4|C_1|^2 = J^2(4|C_1|^2 - B_1^2)$.

$$\Gamma_S = \frac{B_1 \pm J\sqrt{(4|C_1|^2 - B_1^2)}}{2C_1} \qquad |\Gamma_S|^2 = \frac{B_1^2 + (4|C_1|^2 - B_1^2)}{4|C_1|^2} = 1 \qquad \text{Eq. 9-16}$$

With stability coefficient K < 1, the source and load reflection coefficients become equal to 1 when we try to conjugate match the device. As a consequence, we conclude that conjugate input-output matching for an LNA is possible only with an unconditionally stable LNA.

Gain Definition in Conjugately Matched Conditions: Maximum Available Gain

We have demonstrated that conjugate matching is possible only with the stability coefficient K > 1. In that case ($\Gamma_S = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$), the transducer gain $G_T = |S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2 / |(1 - S_{11}\Gamma_S)(1 - \Gamma_{OUT}\Gamma_L)|^2$ is the same as the available gain:

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{(1 - S_{11}\Gamma_S)^2(1 - |\Gamma_{OUT}|^2)} = G_{AV} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{(1 - S_{11}\Gamma_S)^2\left(1 - \dfrac{|S_{22} - \Delta S\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}\right)} \qquad \text{Eq. 10-1}$$

$$\text{After calculating GT denominator} \quad G_T = \frac{|S_{21}|^2}{|\Delta S|^2 - |S_{11}|^2 - (\pm)\left(\dfrac{\sqrt{(B_1^2 - 4|C_1|^2)}}{1 - |\Gamma_S|^2}\right)} \qquad \text{Eq. 10-2}$$

Then

$$1 - |\Gamma_S|^2 = \frac{4|C_1|^2 - \left(B_1 + (\pm)\sqrt{B_1^2 - 4|C_1|^2}\right)^2}{4|C_1|^2}$$

becomes

$$1 - |\Gamma_S|^2 = \frac{-2\sqrt{(B_1^2 - 4|C_1|^2)}}{(\pm)B_1 - \sqrt{(B_1^2 - 4|C_1|^2)}} \qquad \text{Eq. 10-3}$$

The above can be replaced by its value in the transducer gain equation, expressed as:

$$G_T = \frac{2|S_{21}|^2}{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2 - (\pm)\sqrt{B_1^2 - 4|C_1|^2}} \qquad \text{Eq. 10-4}$$

We just have to reintroduce the stability coefficient

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2}{2|S_{21}S_{12}|} \geq 1 \qquad \text{Eq. 7-8}$$

and it's associated expression (Eq. 9-11): $B_1^2 - 4|C_1|^2 = 4|S_{12}S_{21}|^2(K^2 - 1)$.

$$G_T = \frac{2|S_{21}|^2}{2|S_{12}S_{21}|K - 2(\pm)|S_{12}S_{21}|\sqrt{K^2 - 1}} = \frac{|S_{21}|}{|S_{12}|}\,\frac{1}{K \pm \sqrt{K^2 - 1}}\; G_T = \frac{|S_{21}|}{|S_{12}|}\left(K \pm \sqrt{K^2 - 1}\right) \qquad \text{Eq. 10-5}$$

The maximum gain you can obtain from an LNA under conjugately matched conditions is called the maximum available gain (MAG). To define MAG we must first calculate the intermediate quantities B_1 and C_1 (Eq. 9-4) as done in section 9:

$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta S|^2$
$C_1 = (S_{11} - S_{22}{}^*\Delta S)$

B_1 should be calculated first, because its polarity determines which sign to use before the radical. For defining Γ_S versus stability coefficient K in section 9, we demonstrated that if B_1 is positive, the "-" sign must be used, and if B_1 is negative the "+" sign must be used. This last rule is also true for the MAG. Note that K must be greater than 1 and ΔS less than 1 (unconditionally stable), or the MAG equation will be undefined. For $K > 1$ the radical in the equation produces an imaginary number, and the MAG calculation is no longer valid. Thus, the MAG is undefined for unstable LNAs.

For the case $K > 1$, we express B_1 as a function of $|C_1|^2$:

$|C_1|^2 = (|S_{11}|^2 - |\Delta S|^2)\,(|1 - S_{22}|^2) + |S_{12}S_{21}|^2$

$$B_1 = \frac{|C_1|^2 + (1 - |S_{22}|^2)^2 - |S_{21}S_{12}|^2}{1 - |S_{22}|^2} \qquad \text{Eq. 10-6}$$

For stable LNAs, S_{22} is less than 1. In section 7, where we listed the equation for the stability coefficient K, we stated that $|\Delta S| < 1$. As a consequence, $(1 - |S_{22}|^2) \geq |S_{21}S_{12}|$. Thus, we can state that $B_1 > 0$ for an unconditionally stable LNA, and therefore the minus sign must be used before the radical. For an unconditionally stable LNA we can express the maximum transducer gain (equal to the maximum available gain) as a function of the stability coefficient.

$$G_{TMAX} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right) \qquad \text{Eq. 10-7}$$

The maximum available gain can then be expressed in dB:

$$MAG = 10LOG\,\frac{|S_{21}|}{|S_{12}|} + 10Log\left(K - \sqrt{K^2 - 1}\right) \qquad \text{Eq. 10-8}$$

For the particular case in which $K = 1$, G_{TMAX} is defined by $G_{MS} = |S_{21}| / |S_{12}|$. This is the maximum stable gain.

For the case in which K and ΔS are simultaneously more than 1, the device input and output can be simultaneously matched but it will be potentially unstable. $B_1 < 1$ for this case, so the plus sign must be used before the radical.

$$G_{TMAX} = \frac{|S_{21}|}{|S_{12}|}(K + \sqrt{K^2 - 1})$$

Design for a Specified Gain

When designing amplifiers, it is often required that a single stage provide a certain amount of gain—no more, and no less. For that situation, a simultaneous conjugate match for LNA would probably provide too much gain for the stage and would probably overdrive its load (or the succeeding stage). You can search through mountains of literature if you wish, hoping to find an LNA that provides exactly the amount of gain needed when conjugately matched. This approach could take weeks or even months, however. Even if you find an LNA with the gain needed, you are at the mercy of the manufacturer and subject to gain variations among LNAs of the same type. A better way that easily alleviates these problems is called selective mismatching.

Selective mismatching is a controlled and manageable way to limit gain by not matching the LNA to its load. Though it may sound like heresy to some, that technique is supported by a practical, logical, and well-accepted design procedure. Some still believe that an LNA must be matched to its source and load impedances at RF frequencies, but that isn't true. An LNA should be simultaneously conjugate matched to its source and load only if maximum gain is desired, and without regard for other parameters such as noise figure and bandwidth.

One of the easiest methods for selectively mismatching an LNA is through the use of a constant-gain circle plotted on a Smith chart. The circumference of a constant-gain circle represents the locus of points (load impedances) that force the amplifier gain to a specified value. For example, any of the infinite number of impedances located on the circumference of a 10dB constant-gain circle will force the amplifier stage gain to 10dB. When the circle is drawn on a Smith chart, you can see the load impedances that provide a desired gain.

A constant gain circle is plotted on a Smith chart by performing a few calculations to determine the center of the circle and its radius. Constant-gain circles are extrapolated from two-port network power gain (operating or desired). See Eq. 3-9. The following explains how a given power gain equation is written as a circle equation:

$$\begin{aligned} G_{DESIRED} &= \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - S_{22}\Gamma_L)^2|S_{11} - \Delta S\Gamma_L|^2\Gamma} \\ &= \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |S_{11}|^2 + (|S_{22}|^2 - |\Delta S|^2)|\Gamma_L|^2 - 2\mathrm{Real}[(S_{22} - S_{11}{}^*\Delta S)\Gamma_L]} \end{aligned} \qquad \textbf{Eq. 11-1}$$

To simplify the equations, define $G = G_{DESIRED} / |S_{21}|^2$ and $D_2 = |S_{22}|^2 - |\Delta S|^2$ and $C_2 = (S_{22} - \Delta S S_{11}{}^*)$.

$$G = \frac{(1 - |\Gamma_L|^2)}{(1 - |S_{11}|^2 + D_2|\Gamma_L|^2 - 2\text{Real}(C_2\Gamma_L)} \qquad \textbf{Eq. 11-2}$$

$$|\Gamma_L|^2(1 + GD_2) - G2\text{Real}(C_2\Gamma_L) = 1 - G(1 - |S_{11}|^2) \qquad \textbf{Eq. 11-3}$$

$$|\Gamma_L|^2 - \frac{G2\text{Real}(C_2\Gamma_L)}{(1 + GD_2)} = \frac{1 - G(1 - |S_{11}|^2)}{(1 + GD2)} \qquad \textbf{Eq. 11-4}$$

$$|\Gamma_L|^2 - \frac{G2\text{Real}(C_2\Gamma_L)}{(1 + GD_2)} + \frac{(GC_2)^2}{(1 + GD_2)^2} - \frac{(GC_2)^2}{(1 + GD_2)^2} = \frac{1 - G(1 - |S_{11}|^2)}{(1 + GD2)} \qquad \textbf{Eq. 11-5}$$

$$|\Gamma_L|^2 - \frac{G2\text{Real}(C_2\Gamma_L)}{(1 + GD_2)} + \frac{(GC_2)^2}{(1 + GD_2)^2} = \frac{(GC_2)^2 + [1 - G(1 - |S_{11}|^2)](1 + GD2)}{(1 + GD2)^2} \qquad \textbf{Eq. 11-6}$$

$$\left|\Gamma_L - \frac{(GC_2^*)}{(1 + GD_2)}\right|^2 = \frac{(GC_2)^2 + [1 - G(1 - |S_{11}|^2)](1 + GD2)}{(1 + GD2)^2} \qquad \textbf{Eq. 11-7}$$

$$\left|\Gamma_L - \frac{(GC_2^*)}{(1 + GD_2)}\right|^2 = \frac{1 - G(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta S|^2) + G^2(C_2^2 - D_2 + D_2|S_{11}|^2)}{(1 + GD_2)^2} \qquad \textbf{Eq. 11-8}$$

The radius for a constant-gain circle can be written as a function of the stability coefficient K. After rearrangement, we have a constant-gain circle equation:

$$\left|\Gamma_L - \frac{(S_{22}^* - \Delta S^* S_{11})G}{(1 + G(|S_{22}|^2 - |\Delta S|^2)}\right|^2 = \frac{1 - 2K|S_{12}S_{21}|G + |S_{12}S_{21}|^2G^2}{(1 + G(|S_{22}|^2 - |\Delta S|^2)} \qquad \textbf{Eq. 11-9}$$

The Constant Gain Circles coordinates can be expressed as:

$$\text{Center Location} = \frac{(S_{22}^* - \Delta S^* S_{11})G}{(1 + G(|S_{22}|^2 - |\Delta S|^2)} \qquad \text{Radius} = \frac{\sqrt{1 - 2K|S_{12}S_{21}|G + |S_{12}S_{21}|^2G^2}}{(1 + G(|S_{22}|^2 - |\Delta S|^2)} \qquad \textbf{Eq. 11-10}$$

Eq. 11-10 produces a complex number for the center location in magnitude-angle format, similar to that of a reflection coefficient. That number is plotted on the chart exactly as you would plot a reflection-coefficient value. The radius of the circle calculated with Eq. 11-10 is simply a fractional number between zero and one, which represents the size of that circle with respect to a Smith chart. A circle of radius "one" has the radius of the Smith chart, a radius of "0.5" represents half the radius of the Smith chart, and so on.

When you choose the load-reflection coefficient (and therefore the load impedance you will use), the next step is to determine the value of source-reflection coefficient needed to complete the design without producing a further decrease in gain. This value is the conjugate of the LNA's actual input reflection coefficient (with specified load), and is given by Eq. 9-2.

In the Smith chart below (**Figure 11-1**), the circles represent different constant gains (desired gains) that can be obtained with the same LNA. The circle with largest radius represents the lowest constant-gain value. The circle with zero radius represents the maximum available gain condition, in which the LNA input and output are conjugately matched. All circle centers are located on the same axis.

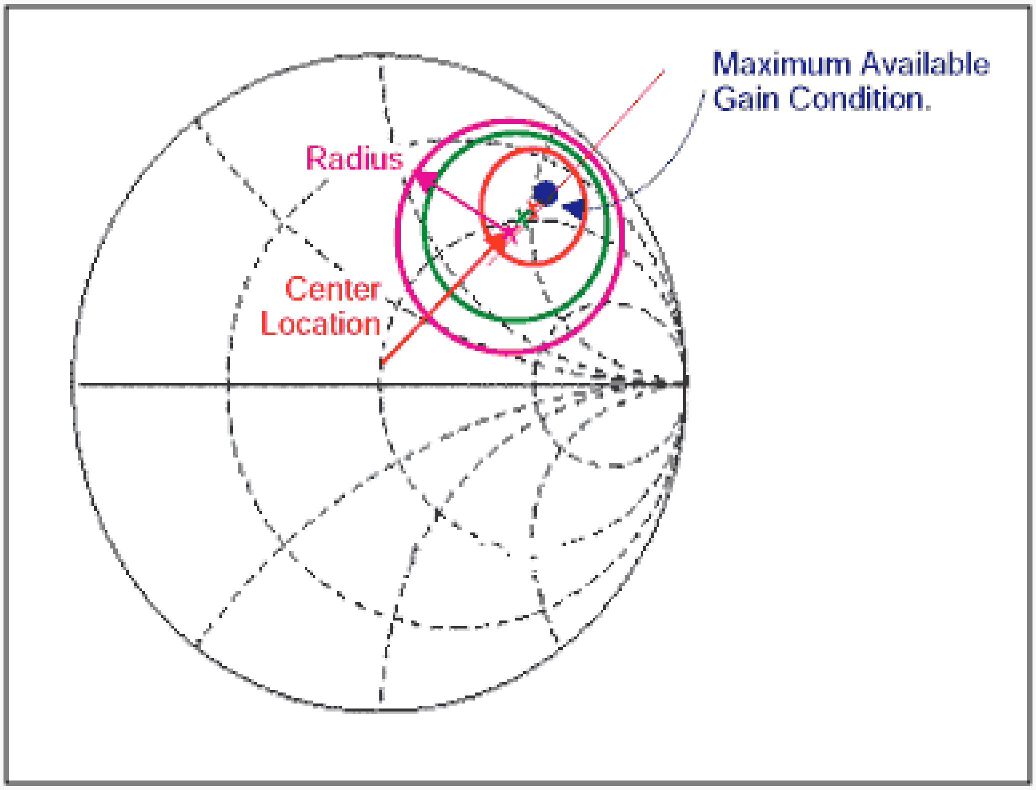


Figure 11-1.

Reference

Bowick, Chris. *RF Circuit Designs*. Howard W. Sams & Co. Inc., a publishing subsidiary of ITT.

Related Parts

MAX2320: QuickView -- Full (PDF) Data Sheet -- Free Samples

Automatic Updates

Application note 1851: www.maxim-ic.com/an1851

More Information

AN1851, AN 1851, APP1851, Appnote1851, Appnote 1851

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APPLICATION NOTE 1852

Low-Noise Amplifier Stability Concept to Practical Considerations, Part 3

Abstract: Part 3 completes the series by presenting application examples. The first shows how to match an LNA in the maximum available gain condition. The second deals with an LNA matched in the constant desired gain condition. The third exercise stresses the importance of matching a potentially unstable LNA in its stable area.

- Part 1
- Part 2
- Part 3

In Part 1, we started our discussion with a brief background on transmission lines and a reminder about RF power gain definitions. Next, in Part 2 we jumped into the RF aspect of low-noise amplifiers by examining stability (tendency for oscillation), impedance matching, and general amplifier design, using scattering parameters (S-parameters) as design tools. Part 3 completes the series by presenting application examples. The first shows how to match an LNA in the maximum available gain condition. The second deals with an LNA matched for constant gain at a desired level. The third exercise stresses the importance of matching a potentially unstable LNA in its stable area.

Applications

To support the theory of LNA matching with practical examples, we turn first to the MAX2720/MAX2721 PA drivers (**Figure 1**). These are low-cost, high-performance, direct I/Q modulators designed for use in WCDMA and wireless-local-loop (WLL) systems. When compared with devices featuring dual-conversion architectures, their direct up-conversion architecture reduces the system cost, component count, and board space.

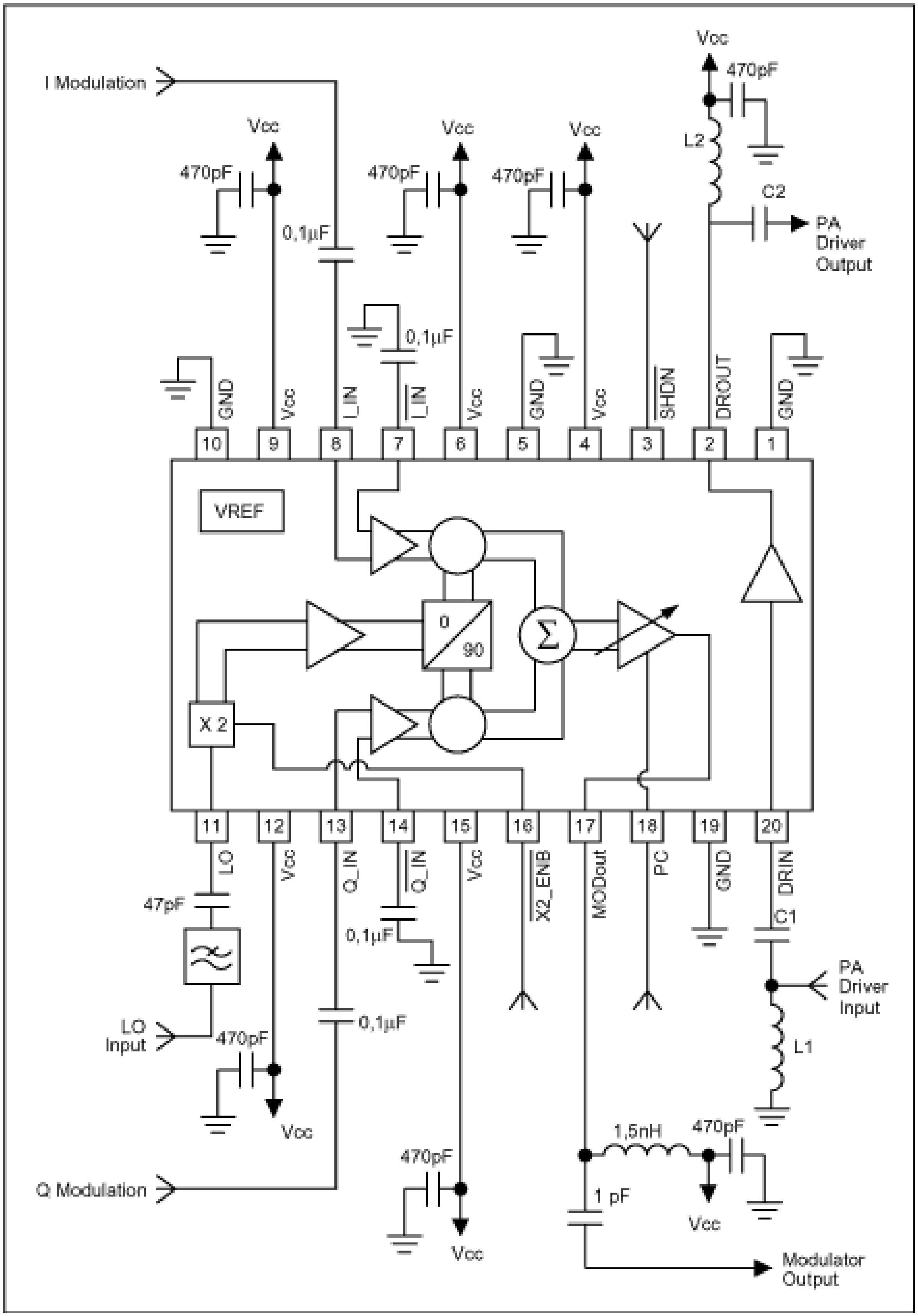

Figure 1. Typical operating circuit for the MAX2721 PA driver.

The MAX2720 and MAX2721 include an I/Q modulator, a variable gain amplifier (VGA), and a power amplifier (PA) driver. The quadrature modulator accepts differential baseband I/Q signals and directly modulates an RF carrier in the 1.7GHz to 2.1GHz range (MAX2720) or the 2.1GHz to 2.5GHz range (MAX2721).The first application example features a MAX2721 PA driver used in the maximum-available-gain condition at 2.3GHz. The PA driver must operate between 50Ω terminations. As explained in chapter 10, this requirement means that the MAX2721's input and output matching networks must meet a simultaneous conjugate-match condition. The

MAX2721 has the following S-parameters (magnitude/angle) at 2.3GHz:

S_{11} = 0.456/135.8°
S_{21} = 3.176/80.4°
S_{12} = 0.11/75.9°
S_{22} = 0.051/-133.9°

The PA driver is usable in the maximum-available-gain mode only if unconditionally stable at the 2.3GHz operating frequency. Before beginning any design calculation, first use Eqs.7-8 and 7-9 to check whether the stability coefficient K is larger than one.The intermediate value is:
Magnitude $|\Delta S| = |S_{11}S_{22} - S_{21}S_{12}|$ = 0.370. Angle = -22.146°

Use the intermediate value to calculate $K = \dfrac{1 - |0.456|^2 - |0.051|^2 + |0.37|^2}{2*|3.176*0.11|}$ = 1.326.

Because K is larger than 1 and |ΔS| less than 1, the PA driver is unconditionally stable, and we may proceed. The maximum available gain is given by Eq. 10-8:

$$\mathrm{MAG} = 10\mathrm{LOG}\frac{|3.176|}{|0.11|} + 10\mathrm{Log}(1.326 - \sqrt{1.326^2 - 1}) = 11.185\mathrm{dB}$$

An external PA driver would be needed if the design specification called for a minimum gain larger than 11.185dB. We will consider 11.185dB adequate for our purpose. The next step is to find the load reflection coefficient needed for a conjugate match.

First, the two intermediate quantities C_2 and B_2 must be found. From Eq. 9-7,

C_2 = (0.051/-133.9°) - (0.456/-135.8°)(0.370/–22.146°) = 0.124/12.416°.

Also from Eq. 9-7,

$B_2 = 1 + (0.051)^2 - (0.456)^2 - (0.370)^2 = 0.657$.

Thus, the magnitude of the load-reflection coefficient can be found using Eq. 9-12. Because $B_2 > 0$, the minus sign can be used before the radical in the reflection coefficient:

$$\Gamma_L = \frac{0.657 - 2|3.176 \bullet 0.11|\sqrt{(1.326^2 - 1)}}{2(0.124)} = 0.196$$

The angle of the load reflection coefficient is equal to the negative of the angle of C_2:

Γ_L = 0.196/-12.416°

The next step is to find the source-reflection coefficient needed for a conjugate match. First, the two intermediate quantities C_1 and B_1 must be found. From Eq. 9-4,

C_1 = (0.456/135.8°) - (0.051/133.9°)(0.370/–22.146°) = 0.439/136.805.

Also from Eq. 9-4,

$B_1 = 1 + (0.456)^2 + (0.051)^2 - (0.370)^2 = 1.068$.

Therefore, the magnitude of the source-reflection coefficient can now be found using Eq. 9-13. Because $B_1 > 0$, the minus sign can be used before the radical in the reflection coefficient:

The angle of the source reflection coefficient equals the negative of angle C_1:

Γ_S = 0.523/-136.805°

Once the desired Γ_S and Γ_L are known, all that remains is to surround the PA driver with components that provide source and load impedances that "look like" Γ_S and Γ_L. Design of the input matching network is shown on the Smith Chart in **Figure 2**. The object of the design is to force the 50Ω source to present a reflection coefficient of 0.523/-136.805°.

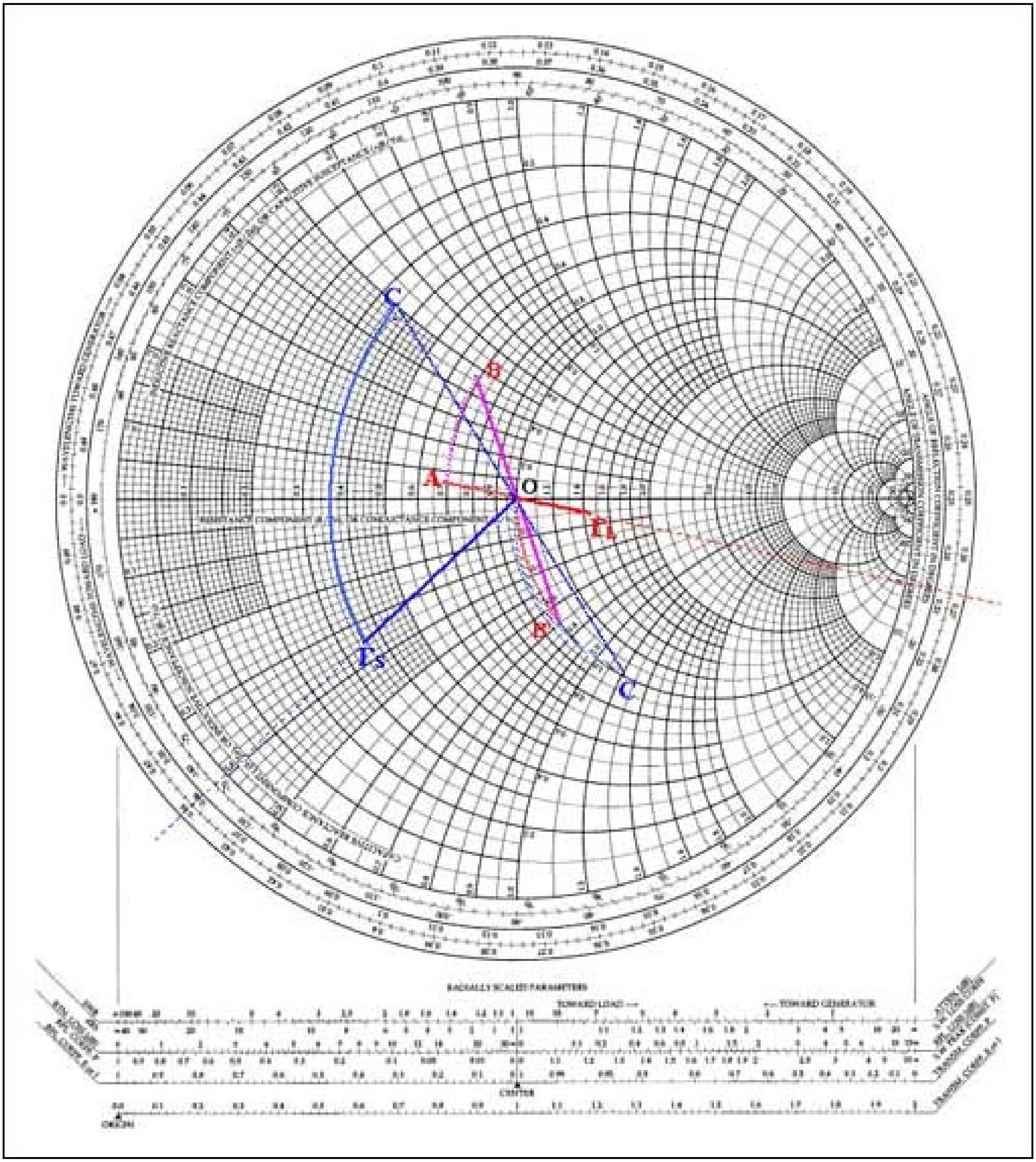

Figure 2. Smith Chart illustrating the input and output impedance matching necessary for a MAX2721 PA driver to operate with maximum available gain.

With Γ_S plotted in blue as shown, the desired, normalized, corresponding impedance is read directly from the chart as Z_S = 0.36 – J0.36Ω. Remember, the impedance is normalized because the chart has been normalized to 50Ω. The actual impedance represented by Γ_S equals 50(0.36 – J0.36)Ω = 18 – J18Ω. To force the 50Ω source to appear as an 18 – J18Ω impedance to the PA driver, we merely add a shunt and a series reactive component as shown on the chart in Figure 2. Proceeding from the source, we have a shunt inductance (L_1), so we have to start in the admittance chart.

Start from zero on the Smith Chart, which is the normalized 1(1/50Ω) source admittance, and turn counter-clockwise, because inductance in shunt produces the negative admittance of a quantity yet to be determined. Seen from the PA driver input on the plot, the end point must be situated on a circle of constant r = 1, such that the C_1 serial capacitor (counter-clockwise rotation in the impedance chart) will end at the point Γ_S.

Arc OC gives the value of the shunt inductance L_1.
Arc C'Γ_S gives the value of the serial capacitor C_1.

The measured value of the arc OC is 1.2 units, so Z = 50/1.2 = 41.66Ω.
Thus, L_1 = 41.66/ω = 41.66/2π f = 41.66/2π (2.3 × 10^9) = 2.88nH, rounded to 3nH. The measured value of the arc C'Γ_S is 0.86 units, so Z = 50 × 0.86 = 43Ω.
Thus, C_1 = 1/(43ω) = 1/(43 × 2π f) = 1/(43 × 2π × 2.3 × 10^9) = 1.609pF, rounded to 1.6pF. This completes the input-matching network.

The load-reflection coefficient is plotted in red on the Smith Chart (Figure 2). As read from the chart, it represents a desired load impedance of Z_L = 50(01.47 -J0.13)Ω = 73.5 – J6.5Ω. The matching network is designed as follows, proceeding from the load:

On the Smith Chart start from zero, which is the normalized 1(50Ω) load impedance. Turn counter-clockwise, because the serial capacitor C_2 implies the negative reactance of a quantity yet to be determined. The end point must be situated on the circle of constant r = 1, so the L_2 shunt inductance (counter-clockwise rotation in the admittance chart) ends at point A on the plot.

Arc OB' gives the value of the serial capacitor C_2.
Arc AB gives the value of the shunt inductance L_2.

The measured value of arc OB' is 0.7 units, so Z = 50 × 0.7 = 35Ω.
Thus, C_2 = 1/(35ω) = 1/(35 × 2π f) = 1/(35 × 2π × 2.3 × 10^9) = 1.977pF, rounded to 2pF. The measured value of arc AB is 0.4 units, so Z = 50/0.4 = 125Ω. Thus, L_2 = 125/ω = 125/2π f = 125/(2π × 2.3 × 10^9) = 8.649nH, rounded to 8.2nH.

On the MAX2721 block diagram, note that the 470pF capacitor between ground and the junction of V_{CC} and L_2 is considered a short circuit at 2.3GHz. This completes the output-matching network.

Constant-Gain Application

A second application employs the MAX2721 PA driver in an 8dB constant-desired-gain condition at 2.3GHz. The PA driver must operate between 50Ω terminations. As explained in chapter 11, we match the PA driver input and output impedances by defining a constant-gain circle on the Smith Chart. The PA driver is unconditionally stable with K = 1.326.

Using Eq. 11-10 and proceeding "by the numbers," we locate the center of the circle at a point:

$$\text{Center Location} = \frac{(0.051/133.9^\circ - 0.37/22.14^\circ \bullet 0.466/135.8^\circ)6.31}{(1 + 6.31(|0.051|^2 - |0.370|^2)}$$

Center is therefore located at 0.085/-12.416°.

This point can now be plotted on the Smith Chart. The radius of the 8dB-gain circle is calculated as:

$$\text{Radius} = \frac{\sqrt{1 - 2 \bullet 1.326 \bullet |0.11 \bullet 3.176| \bullet 6.31 + |0.11 \bullet 3.176|^2 \bullet 6.31^2}}{(1 + 6.31(|0.051|^2 - |0.370|^2)}$$

Radius is therefore 0.747.

The Smith Chart construction is shown in **Figure 3**. If the input impedance of the PA driver is conjugately matched, note that any load impedance located along the circumference of this circle produces an amplifier gain of 8dB. As noted in the problem statement, the actual load impedance we have to work with is 50Ω. Its normalized value is shown as the Smith Chart origin (point O) in Figure 3. The PA driver's output network must transform the actual load impedance into a value that falls on the constant gain circle.

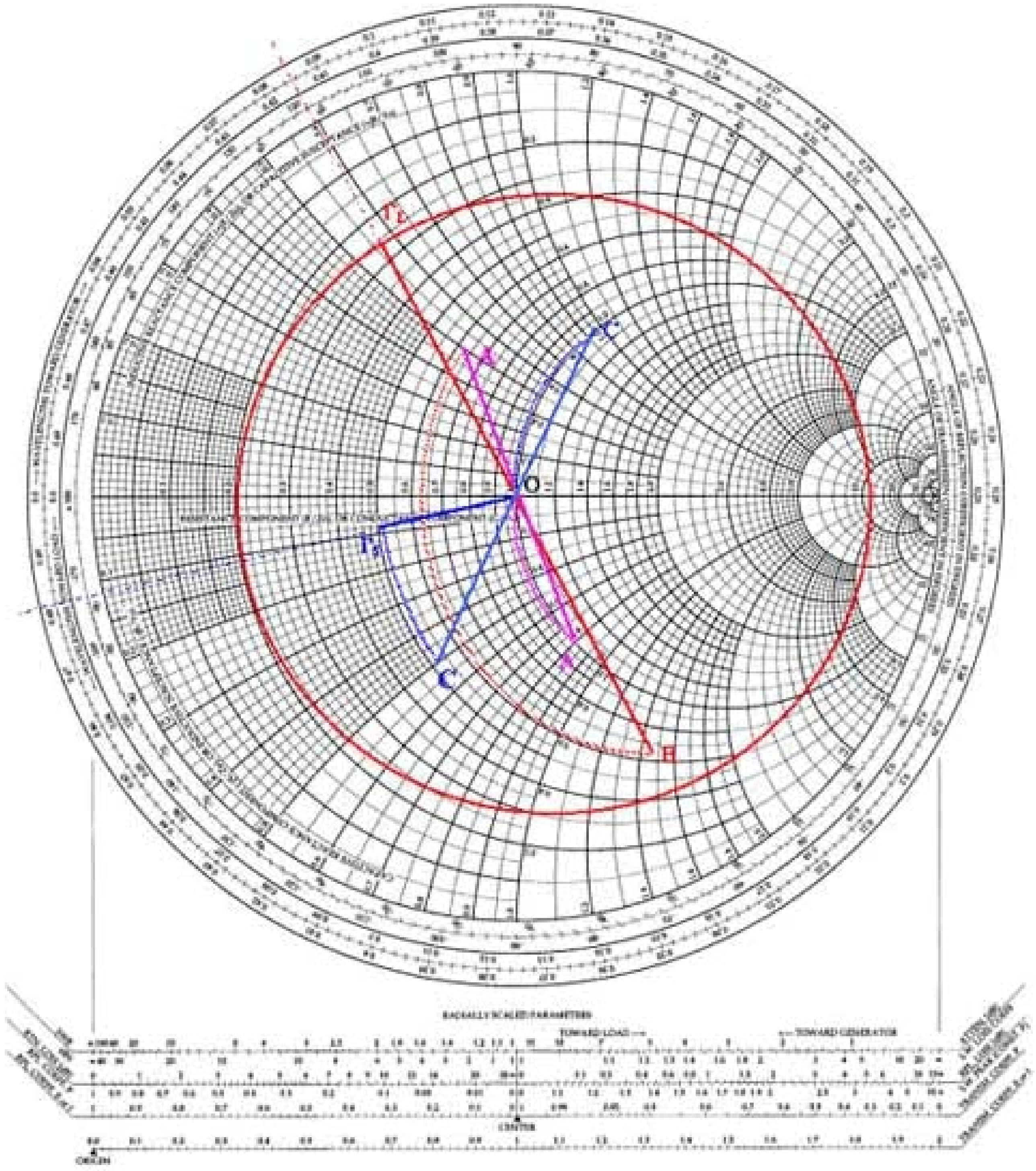

Figure 3. Smith Chart illustrating the input and output impedance matching necessary for a MAX2721 PA driver to operate with a specified gain (8dB in this case).

Several circuit configurations can accomplish this task. For convenience, we choose the configuration $\Gamma_L = 0.68/+118°$ for the load reflection coefficient (shown in red on the Smith Chart). Proceeding from the load,

Arc A'B gives the value of shunt inductance L_1 (counter-clockwise in the admittance chart).
Arc OA gives the value of series capacitor C_1 (counter-clockwise in the impedance chart).

The measured value of arc A'B is 1.9 units, so $Z = 50/1.9 = 26.31\Omega$.
Thus, $L_2 = 26.31/\omega = 26.31/(2\pi f) = 26.31/(2\pi \times 2.3 \times 10^9) = 1.82\text{nH}$, rounded to 1,8nH. The measured value of arc OA is 0.8 units, so $Z = 50 \times 0.8 = 40\Omega$.
Thus, $C_2 = 1/(40 \times \omega) = 1/(40 \times 2\pi f) = 1/(40 \times 2\pi \times 2.3 \times 10^9) = 1.8\text{pF}$, rounded to 1.8pF.

For a conjugate match at the input to the PA driver, with $\Gamma_L = 0.68/+118°$, the desired source reflection coefficient must be as shown in Eq. 9-1:

$$\Gamma_S = \left(\frac{[0.456/135.8^{\circ} - 0.37 - 22^{\circ} \bullet 0.68/118^{\circ}]^*}{[1 - 0.051/-133.9^{\circ} \bullet 0.68/118^{\circ}]^*} \right)$$

The source reflection coefficient Γ_S is therefore 0.319/-167°.

This point is plotted as Γ_S in Figure 3. As given in the problem statement, the actual source impedance is 50Ω. Its normalized value is shown as the Smith Chart origin (point O). Thus, the input network must transform the actual impedance at point O to the desired impedance at point Γ_S. For practice, this was done with a two-element design as shown.

Arc Γ_SC' gives the value of series capacitor C_1 (counter-clockwise in the impedance chart).
Arc OC gives the value of shunt inductance L_1 (counter-clockwise in the admittance chart).

The measured value of arc Γ_SC' is 0.42 units, so $Z = 50 \times 0.42 = 21\Omega$.
Thus, $C_2 = 1/(21 \times \omega) = 1/(21 \times 2\pi f) = 1/(21 \times 2\pi \times 2.3 \times 10^9) = 3.29\text{pF}$, rounded to 3.3pF. The measured value of arc OC is 0.93 units, so $Z = 50/0.93 = 53.8\Omega$.
Thus, $L_2 = 53.8/\omega = 53.8/(2\pi f) = 53.8/(2\pi \times 2.3 \times 10^9) = 3.72\text{nH}$, rounded to 3.8nH.

Match for Stability

In a third application, the MAX2320 PCS-Band LNA (**Figure 4**) operates at 13.5dB gain as described in the MAX2320 data sheet under high-gain, high-linearity mode of operation. The MAX2320 (a high-performance, silicon-germanium (SiGe) receiver front-end) is one of a 6-member family of ICs dedicated to dual-band, triple-mode CDMA/TDMA/GSM handset applications. These devices set a new standard for low noise and high linearity at low supply current. They integrate a variety of features, including an LO frequency doubler and divider, gain settings for a dual low-noise amplifier (LNA), and a low-current paging mode that extends the handset standby time.

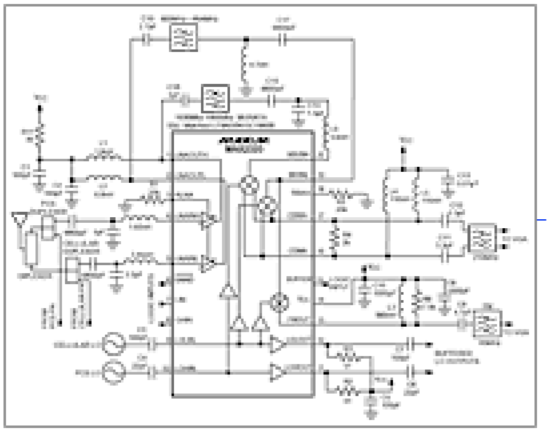

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Figure 4. Typical operating circuit for the MAX2320 PCS-band LNA (receiver front end).

As a 50Ω termination, the LNA output is loaded by a Murata bandpass filter of 1960MHz ± 30MHz (LFSN30N15C1960). For matching the LNA input and output, we define on the Smith Chart a circle of constant gain (13.5dB) as explained in chapter 11. The MAX2320 has the following S-parameters (magnitude/angle) at 1.95GHz:

S_{11} = 0.43/-115°

S_{21} = 3.82/84°

S_{12} = 0.09/75°

S_{22} = 0.673/-57°

We locate the center of the circle at the point:

$$\text{Center Location} = \frac{(0.673/-57^\circ - 0.167/-78.12^\circ \cdot 0.43/115^\circ)22.387}{(1 + 22.387(|0.673|^2 - |0.167|^2)}$$

The center is therefore located at 0.633/+63°.

On the Smith Chart (Figure 5), the radius of this 13.5dB-gain circle is calculated as:

$$\text{Radius} = \frac{\sqrt{1 - 2 \cdot 0.567 \cdot |0.09 \cdot 3.82| \cdot 22.387 + |0.09 \cdot 3.82|^2 \cdot 22.387^2}}{(1 + 22.387(|0.673|^2 - |0.167|^2)}$$

The radius is therefore 0.5.

A calculation of the stability factor K for the MAX2320 PCS-Band LNA indicates a potential instability at K = 0.567 and ΔS = 0.167. To prevent oscillation, we must therefore exercise extreme caution in choosing source and load impedances for the device. To find stable operating regions on the Smith Chart (**Figure 5**), we must plot the input and output stability circles. Because |ΔS| is less than $|S_{11}|$ or $|S_{22}|$, the areas of stability are outside the input and output stability circles.

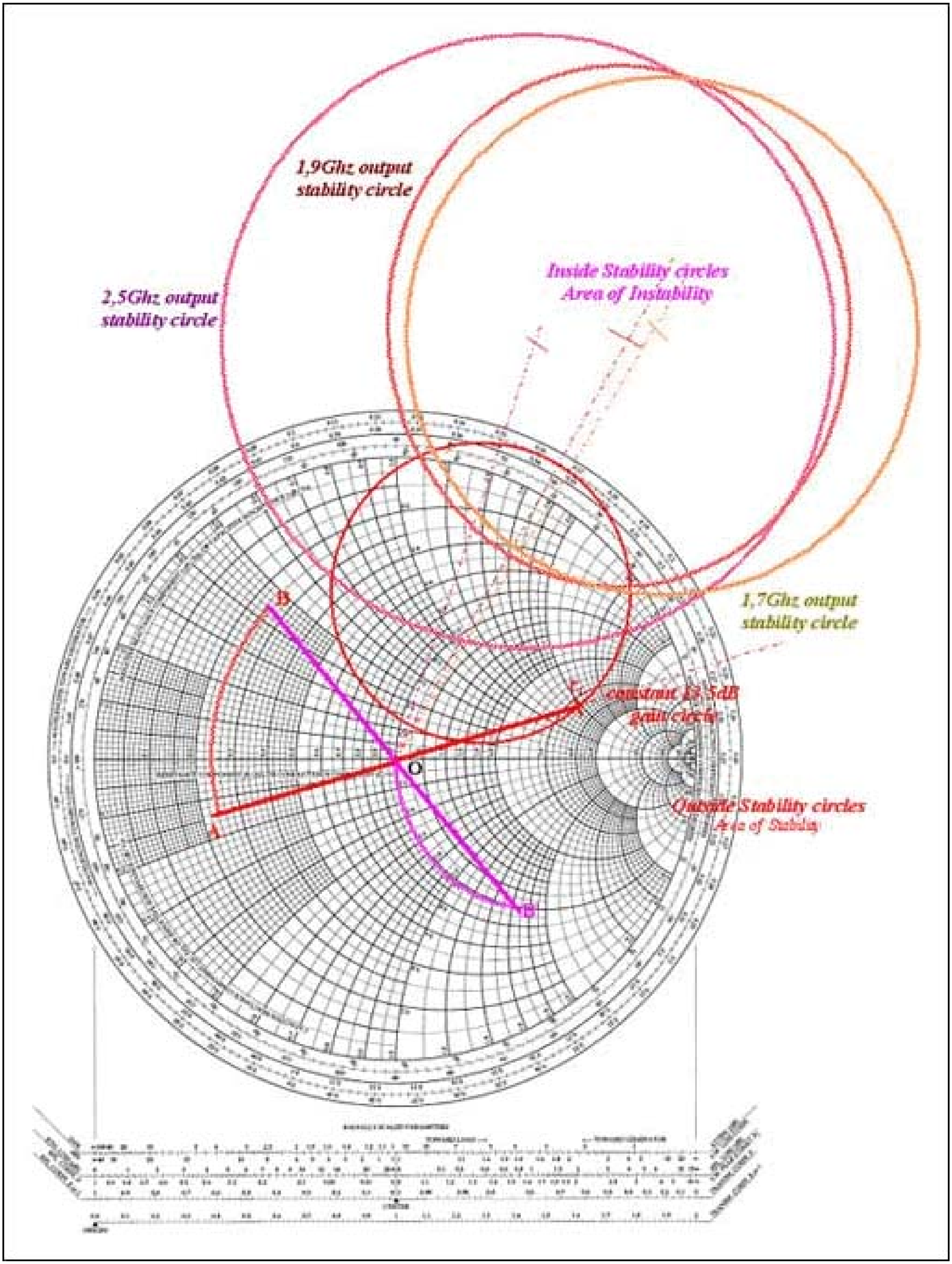

Figure 5. The output of this MAX2320 PCS LNA is matched for a desired gain of 13.5dB.

The only load impedance that we may not select for the LNA is located inside the output stability circle. Any other load impedance located on the 13.5dB gain circle will provide the needed gain, provided the device input is conjugately matched, and provided the input reflection coefficient Γ_S required for a conjugate match falls outside the input stability circle.

We choose load reflection coefficient $\Gamma_L = 0.6/18°$ as a convenient value on the 13.5dB gain circle.

Arc Γ_LA gives the value of the shunt inductance L_2 (counter-clockwise in the admittance chart).
Arc B'O gives the value of the series capacitor C_2 (counter-clockwise in the impedance chart).

The measured value of arc Γ_LA is 0.56 units, so Z = 50/0.56 = 89.29Ω.

Thus, $L_2 = 89.29/\omega = 89.29/(2\pi\ f) = 89.29/\ (2\pi \times 1.96 \times 10^9) = 7.25$nH, rounded to 7.2nH. The measured value of arc B'O is 1.7 units, so Z = 50 × 1.7 = 85Ω.
Thus, $C_2 = 1/(85 \times \omega) = 1/(85 \times 2\pi\ f) = 1/(85 \times 2\pi$ /appnotes/1852$\times\ 1.96 \times 10^9) = 0.955$pF, rounded to 1pF.

Using Eq. 9-1, calculate the source reflection coefficient Γ_S needed for a conjugate match, and plot that point on the Smith Chart (**Figure 6**):

$$\Gamma_S = \left(\frac{[0.43/-115^\circ - (0.167/-78.12^\circ \bullet 0.6/+18^\circ)]^*}{[1-(0.673/-57^\circ \bullet 0.6/+18^\circ)]^*}\right)$$

Source Reflection Coefficient: $\Gamma_S = 0.521/+147.74°$. Note that Γ_S falls in the stable region of the input stability circle (Figure 6), and therefore represents a stable termination for the LNA.

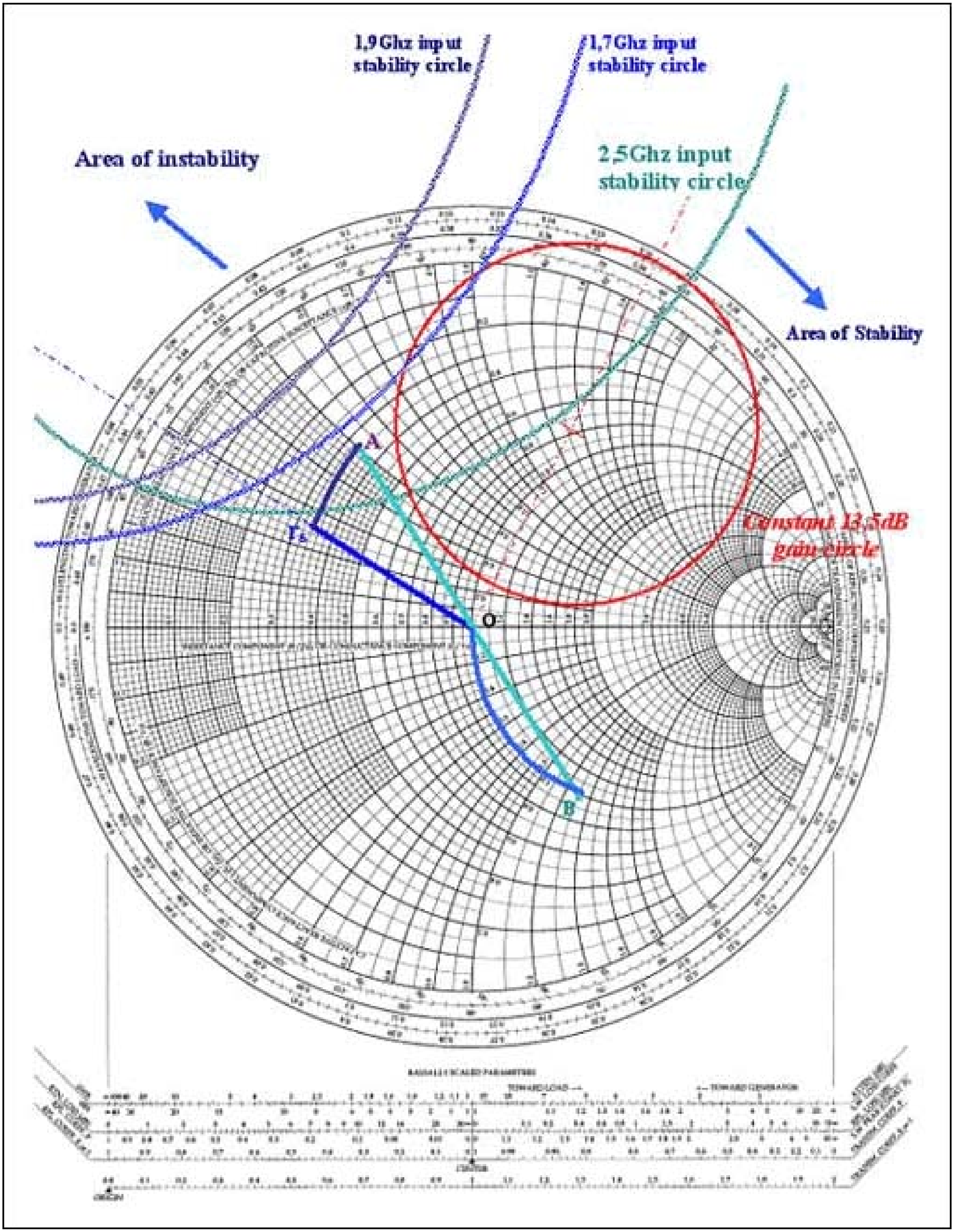

Figure 6. The input of this MAX2320 PCS LNA is matched for a desired gain of 13.5dB.

The 6800pF high-value capacitor between the duplexer and inductance L_1 is not to be considered in the input-matching calculation, because it assumes only a DC-blocking function for the LNA input. Loaded with a 50Ω antenna, the PCS duplexer shows a 50Ω impedance that must be matched to the LNA input.

Arc $\Gamma_S A$ gives the value of series inductance L_1 (clockwise in the impedance chart).
Arc BO gives the value of shunt capacitor C_1 (clockwise in the admittance chart).

The measured value of arc $\Gamma_S A$ is 0.22 units, so $Z = 50 \times 0.22 = 11\Omega$.

Thus, $L_1 = 11/\omega = 11/(2\pi\ f) = 11/(2\pi \times 1.96 \times 10^9) = 0.893nH$, rounded to 1nH. The measured value of arc BO is 1.3 units, so $1/Y = Z = 50/1.3 = 38.46\Omega$.
Thus, $C_1 = 1/(38.46 \times \omega) = 1/(38.46 \times 2\pi\ f) = 1/(38.46 \times 2\pi \times 1.96 \times 10^9) = 2.11pF$, rounded to 2.2pF.

Reference

Bowick, Chris. *RF Circuit Designs*. Howard W. Sams & Co. Inc., 4300 West 62nd Street, Indianapolis, Indiana (a publishing subsidiary of ITT).

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