Residual amplitude modulation from electro-optic phase modulation

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1 Introduction

This brief document is meant to elucidate the origins of residual amplitude modulation (RAM) present when phase-modulating an optical beam with an electro-optic modulator (EOM), and to highlight methods for reducing it.

2 The EOM

An EOM essentially consists of a crystal exhibiting the Pockels effect (i.e., a birefringence linearly dependent on an applied electric field), and electrodes to apply the external field. In the phase modulation (PM) configuration, a linearly polarized optical beam is transmitted through the crystal with its polarization aligned with the axis of birefringence modulation. When a signal is applied to the EOM, it modulates the phase velocity of the light, and this results in proportional phase modulation of the output beam. Ideally, the amplitude of the output beam is *not* amplitude-modulated; in practice, this is never the case.

3 RAM

Frequency modulation spectroscopy techniques (like the Pound-Drever-Hall method) operate on the principle of PM-to-AM conversion upon the interaction of an optical field with some sample with a complex coupling factor.

PM sidebands are applied to the optical field at a frequency spacing substantially greater than the interaction linewidth of the sample, so that the coupling factor as seen by the sidebands is purely real whenever the carrier field is near resonance. In this way, the phase shift induced by the sample on the carrier causes the sum of the beat fields produced by the carrier and each of the sidebands respectively no longer to cancel exactly. The result, upon demodulating the post-interaction photocurrent at the PM frequency, is a linear signal usable to lock the carrier frequency to the resonance frequency of the sample.

It is straightforward to see that any AM existing at the PM frequency *a priori* will result in an offset in the locking servo loop. To the extent that it is not large enough to push the system out of the linear operating range of the interaction, a constant AM level is not necessarily crippling to an experiment. Intentional offsets can also be added into the loop to null the effect of a constant AM level. The main problem with RAM is that it tends not to be constant in time. This drift in the AM level appears to the system in *exactly the same way* as a signal. It is typical for low-frequency AM drifts due to thermal effects to limit the performance of spectroscopic experiments at low frequencies. For this reason, it is nice to know how RAM comes about, and how to alleviate its impact in each case.

4 Origins

RAM results in at least three ways, two of which are more-or-less fully understood, and the other not at all.

4.1 Polarization misalignment

If the polarization axis of the input beam is misaligned with that of the modulation axis, the phase modulation from the crystal results in a change in the polarization vector of the light. In general, the output of the beam will be elliptically polarized, and any polarizing elements later in the optical train will result in amplitude modulation.

To see how this happens, consider the following, typical optical setup: An S-polarized (linear, vertical) beam is fed into an EOM for the purpose of phase modulation. At the output, it is desired to split the beam into two beams of equal power so as to send them to different sections of an experiment. This is accomplished by using a half-wave plate (HWP) to rotate the linear polarization of the beam emerging from the EOM 45° clockwise, then passing the beam through a polarizing beamsplitter (PBS), which splits the power evenly between the S- and P-polarized components.

We would like to slightly complicate the above situation by allowing the input beam to make a small angle α with the vertical. Using the standard Jones formalism, normalizing the input amplitude to unity and neglecting the common optical phase factor, the input beam is described by the vector

$$\vec{E}_0 = \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}. \tag{1}$$

The EOM, with its axis oriented vertically, is described by the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi(t)} \end{pmatrix}, \tag{2}$$

with $\phi(t)$ the time-dependent phase shift applied along the crystal axis. In practice, the orientation of the HWP will be chosen such that the DC power out of each port of the PBS is equal. If the PBS is oriented properly with respect to the coordinate system, then the required rotation angle of the HWP is $(\frac{\pi}{4} - \alpha)$. The matrix describing a HWP that does this is

$$\mathbf{R} = \begin{pmatrix} \cos(\frac{3\pi}{4} - \alpha) & \sin(\frac{3\pi}{4} - \alpha) \\ \sin(\frac{3\pi}{4} - \alpha) & -\cos(\frac{3\pi}{4} - \alpha) \end{pmatrix} \tag{3}$$

Finally, the PBS just selects the S- and P-polarized components of the input beam relative to this coordinate system, so its coupling matrices are

$$\mathbf{P}_S = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

and

$$\mathbf{P}_P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{5}$$

Putting this together, the beam approaching the PBS will have the form

$$\vec{E}_2 = \mathbf{RM}\vec{E}_0 = \begin{pmatrix} \cos(\frac{3\pi}{4} - \alpha)\sin\alpha + e^{i\phi(t)}\sin(\frac{3\pi}{4} - \alpha)\cos\alpha \\ \sin(\frac{3\pi}{4} - \alpha)\sin\alpha - e^{i\phi(t)}\cos(\frac{3\pi}{4} - \alpha)\cos\alpha \end{pmatrix}. \tag{6}$$

Considering power conservation, it is sufficient to examine only one of the beams exiting the beamsplitter. In the case of the transmitted, P-polarized output beam, and assuming $\alpha \ll 1$, we have

$$\left(\vec{E}_p\right)_x \approx \frac{\sqrt{2}}{2}(\alpha - 1)\alpha + e^{i\phi(t)}\left[\frac{\sqrt{2}}{2}(1+\alpha)\right] \approx \frac{\sqrt{2}}{2}(e^{i\phi(t)} + \alpha e^{i\phi(t)} - \alpha), \quad (7)$$

with an intensity

$$\left|\left(\vec{E}_p\right)_x\right|^2 \approx \frac{1}{2}\left[1 + 2\alpha\left(1 - \cos\phi(t)\right)\right]. \quad (8)$$

Amplitude modulation proportional to α is clearly evident.

To understand what this looks like in the typical case of sinusoidal modulation to produce radio-frequency (RF) sidebands on the light, consider a modulation signal of the form

$$\phi(t) = \beta\cos\Omega t + \phi_0. \quad (9)$$

Using the Jacobi-Anger relation, we can expand $\cos\left(\beta\cos\Omega t + \phi_0\right)$ in terms of the Bessel functions of the first kind, $J_n(\beta)$. With the DC phase shift $\phi_0 = 0$, we have

$$\cos\left(\beta\cos\Omega t\right) = J_0(\beta) + 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(\beta)\cos\left(2n\Omega t\right) \quad (10)$$

and there are only even harmonics of the modulation frequency Ω present in the signal; with $\phi_0 = \frac{\pi}{2}$, we have

$$\cos\left(\beta\cos\Omega t + \frac{\pi}{2}\right) = -\sin\left(\beta\cos\Omega t\right) = 2\sum_{n=1}^{\infty}(-1)^n J_{2n-1}(\beta)\cos\left[(2n-1)\Omega t\right] \quad (11)$$

and there are only odd harmonics. For arbitrary ϕ_0, there will in general be even and odd harmonics.

Again, a static AM level from this effect would not be a terrible problem. Unfortunately, thermal variations in the crystal can couple into a changing RAM level in several ways:

- Electro-optic crystals like $LiNbO_3$ exhibit a small, static birefringence on the order of a few per cent even without an external electric field present. This means that, as the crystal changes in length due to thermal fluctuations, there is a changing static phase shift (ϕ_0). As seen above, a change in this phase shift causes the harmonic RF content (and therefore the relative power in any harmonic) of the output beam to fluctuate.

- The birefringence of the crystal itself varies weakly with temperature. This birefringence fluctuation, if of the wrong sign, can exacerbate the effect above.

- Finally, and most straightforwardly, the mechanical system to which the EOM crystal is mounted may itself exhibit measurable thermal expansion, and this can lead to a modulation of the polarization-to-crystal-axis angle α.

4.2 Etalon effect

The EOM is designed to operate with the beam passing through it just once. If this isn't the case, then the exiting beam acquires a complicated, frequency-dependent phase shift and amplitude envelope due to the etalon effect. Since the faces of the crystal must have some finite reflectivity, there is always some multi-pass effect within the crystal. This effect is described in detail by Whittaker et al[1]. Here is a brief version.

Consider an EOM crystal of longitudinal length l, static index of refraction n, and a face power reflectivity of R. Its free spectral range (FSR) is thus $\nu_{FSR} = \frac{c}{2nl}$, where c is the speed of light in vacuum. Apply to it a sinusoidal modulation of frequency Ω, and inject into it an optical field of frequency ω. The roundtrip phase accumulated by a beam circulating within this "cavity" is

$$\phi_{RT} = \phi_0 + \phi_F + \phi_B, \tag{12}$$

where

$$\phi_0 = \frac{\omega}{\nu_{FSR}} \tag{13}$$

is the roundtrip phase without modulation,

$$\phi_F = \beta \sin \Omega t \tag{14}$$

is the modulation-induced phase shift on one forward trip, and

$$\phi_B = \beta \frac{\sin \phi_m}{\phi_m} \sin(\Omega t + \phi_m) \tag{15}$$

is the modulation-induced phase shift on one backward trip. In the above,

$$\phi_m = \frac{\Omega}{2\nu_{FSR}} \tag{16}$$

is the single-pass phase shift of the modulation field.

Given an input field E_0, the field transmitted through the EOM has the form

$$E_{out} = E_0 e^{i(\frac{\phi_0}{2}+\phi_F)} \frac{(1-R)}{1-Re^{i\phi_{RT}}}, \tag{17}$$

with intensity

$$I_{out} = I_0 \frac{(1-R)^2}{1+R^2-2R\cos\phi_{RT}}, \tag{18}$$

where I_0 is the input intensity.

In the limit of small modulation index ($\beta \ll 1$), the above can be expanded as

$$I_{out} \approx I_0 \frac{(1-R)^2}{1+R^2-2R\cos\phi_0}\left[1-\frac{2R\beta\sin\phi_0}{1+R^2-2R\cos\phi_0} f_m(t)\right], \tag{19}$$

where

$$f_m(t) = \left(1+\frac{\cos\phi_m \sin\phi_m}{\phi_m}\right)\sin\Omega t + \left(\frac{\sin{\phi_m}^2}{\phi_m}\right)\cos\Omega t \tag{20}$$

contains the dependence on the modulation. Thus, the overall effect is amplitude modulation of the output intensity, with a phase dependent on Ω and an envelope dependent on the optical frequency ω (implicitly through ϕ_0). This means that even if you are very careful to align the polarization of the input beam to the axis of the crystal, the imperfect anti-reflective surfaces of the crystal will result in persisting amplitude modulation.

This RAM coupling is perhaps the most troubling one, because it allows the spurious signal to feed back on itself: The AM level, and thus the control loop offset, is dependent on the input laser frequency. Typically, the loop control signal might be fed back to the laser frequency control to keep it locked to some fixed reference. This feedback could in principle cause instabilities and prevent the loop from functioning at all. In addition, the etalon effect also leads to fluctuations from the thermal couplings described in the previous section.

4.3 "dc-RAM"

As a solution to the etalon effect problem, Whittaker et al[1] considered transmitting the beam through the EOM crystal at an oblique angle, so as to suppress the circulation of back-reflections. Upon doing so, they quickly discovered yet *another* form of RAM. By observing that this new effect was not dependent on the optical frequency, and that small changes in the input polarization angle did not alter the level of RAM to first order, they concluded that the culprit was not either of the foregoing couplings. This "dc-RAM", as they dubbed it, was evident at the order of $\sim 10^{-5} - 10^{-3}$ relative to the phase modulation level, depending on the crystal used. It also showed a variation of more than an order of magnitude between different crystals of the same type.

The investigators noted that the effect could be made to change sign by changing the modulation phase, but that the phase offset drifted significantly on a timescale of hours.

5 Solutions

Some methods for reducing the impact of RAM were proposed as early as 1981. Here I report on two of these methods.

5.1 2Ω cancellation

Whittaker et al[2] proposed a method in which an additional, phase-locked modulation signal at twice the primary modulation frequency Ω was applied to the EOM, with an amplitude and phase carefully adjusted so as to cancel the 2Ω component of RAM induced in the standard way from the primary modulation signal.

In this way, the experimental readout can be carried out using the PM sidebands at $\pm 2\Omega$, in the absence of RAM. The investigators report an experimental reduction of RAM of a factor of 4.

The downside to this approach is that switching to the second-order sideband readout results in a reduction of sensitivity of $\frac{J_2(\beta)}{J_1(\beta)}$, which can be a substantial factor for small β. This reduction causes a direct loss in shot-noise-limited signal-to-noise ratio (SNR). It is also unclear whether or not the authors suggest this is a solution to the dc-RAM problem.

5.2 DC Bias

Hall et al [3, 4] proposed an adaptation of a method earlier proposed by Stein and Turneaure[5] for use with microwave systems. This method takes advantage of the effect in §4.1: The polarization angle of the beam is *intentionally* misaligned and a controllable DC bias is applied to the electric field across the EOM crystal. By adjusting this DC offset, and presumably the phase of the RF modulation signal, the overall RAM as measured by an out-of-loop photodiode can be minimized with a standard servo.

References

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