

## Characterizing Phase Noise

*A tutorial for the novice RF engineer on how to characterize phase noise*

**By Mini-Circuits Inc.**

The term phase noise is used to describe phase fluctuations due to the random frequency fluctuations of a signal. Phase noise can be caused by a number of conditions, but is mostly affected by an oscillators frequency stability.

An oscillators frequency stability is defined as the measure of the degree to which an oscillator maintains the same value of frequency over a given time. It may be specified in a number of ways with three of the most common terms described here.

An ideal sinewave oscillator may be described by:

$$V(t) = V_0 \sin 2\pi ft$$

In this equation,  $V_0$  is the nominal amplitude of the signal, and  $f$  is the nominal frequency of oscillation.

The instantaneous output of an oscillator may be represented by:

$$V(t) = V_0 [1 + A(t)] \sin \{2\pi ft + \theta(t)\}$$

where  $A(t)$  and  $\theta(t)$  represent the amplitude and phase fluctuations of the signal respectively.

The phase term may be random or discrete and can be displayed on a spectrum analyzer (see figure 1). As shown in this figure, there are two types of fluctuating phase terms. The first term, described by the discrete signals, is called "spurious." They appear as distinct components in the spectral density plot. The second term, random in nature, appears as random phase fluctuations and is commonly called "phase noise."

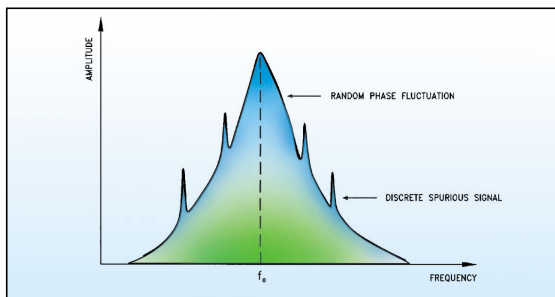


Figure 1. Spectrum analyzer display of phase noise

The source of phase noise in an oscillator is due to "thermal" and "flicker" or  $1/f$  noise. Most oscillators operate in saturation with the AM noise component is usually 20 dB lower than the phase noise component. In the discussion that follows, we will assume that  $A(t) \ll 1$ .

Many methods are used to characterize the phase noise of an oscillator. Essentially, all methods measure the frequency or phase deviation of the source under test, in either the frequency or the time domain. Since frequency and phase are related to each other, all terms are related, as well. One of the most common fundamental descriptions of phase noise is the one-sided "spectral density" of phase fluctuations per unit bandwidth.

Spectral density describes the RMS phase distributions as a continuous function, expressed in units of RMS phase per unit bandwidth. The phase noise of an oscillator is best described in the frequency domain where the spectral density is characterized by measuring the noise "sidebands" on either side of the output signal center frequency. Single-sideband (SSB) phase noise is specified in dBc/Hz at a given frequency offset from the carrier. The frequency domain information about phase or frequency is contained in the power spectral density  $S_{\Delta\theta}(f)$  of the phase or in the power spectral density  $S_{\Delta f}(f)$  of the frequency. Here,  $f$  refers to the modulation frequency or offset frequency associated with the noise-like variations in  $\theta(t)$ .

### Some Basic Relationships

Peak phase modulation  $\Delta\theta$  and peak frequency modulation  $\Delta f$  are related as follows:

$$\Delta\theta_{\text{peak}} = \frac{\Delta f_{\text{peak}}}{f}$$

In terms of RMS values, we have:

$$\Delta\theta_{\text{rms}} = \frac{\Delta f_{\text{rms}}}{f}$$

The one sided spectral distribution of the phase fluctuations per Hz bandwidth is  $S_{\Delta\theta}(f)$ :

$$S_{\Delta\theta}(f) = \frac{(\Delta\theta_{\text{rms}})^2}{BW}$$

In this equation,  $BW$  is the bandwidth of  $\Delta\theta_{\text{rms}}$  measurement. The units of  $S_{\Delta\theta}(f)$  are radian<sup>2</sup> Hz<sup>-1</sup> bandwidth or dB relative to 1 rad<sup>2</sup> Hz<sup>-1</sup> bandwidth. The term  $S_{\Delta\theta}(f)$  is often referred to as the spectral density and describes the energy distribution as a continuous function, expressed in units of energy per Hz bandwidth. This is illustrated in figure 2.

Similarly, one-sided spectral distribution of the frequency fluctuations per Hz bandwidth is  $S_{\Delta f}(f)$  where:

$$S_{\Delta f}(f) = \frac{(\Delta f_{\text{rms}})^2}{BW}$$

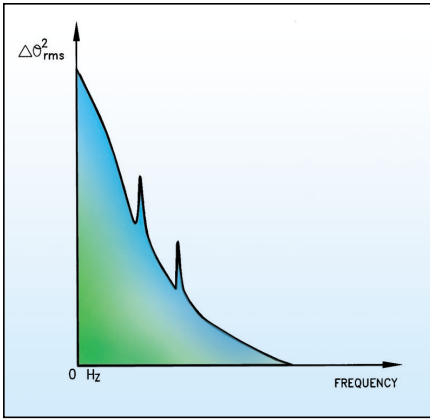


Figure 2. One-sided representation of phase noise spectral density of signal

In this equation,  $BW$  is the bandwidth of  $\Delta f_{rms}$  measurement. The units of  $S_{\Delta\theta}(f)$  are  $(\text{rad sec}^{-1})^2 \text{ Hz}^{-1}$  bandwidth. It is also common to characterize the noise performance of a signal as the ratio of the sideband power associated with phase fluctuations to the carrier power level. If the measure is denoted by  $S_c$ . And,

$$S_c(f) = \frac{\text{power density in one sideband per Hz bandwidth at an offset frequency } f \text{ away from the carrier}}{\text{total signal power}}$$

For small phase fluctuations, we can write:

$$S_c(f) = \left(\frac{\beta}{2}\right)^2$$

Here,  $\beta$  is the modulation index by analogy to standard modulation theory:

$$S_c(f) = \left(\frac{\Delta f_{peak}}{2f}\right)^2 = \frac{(\Delta f_{rms})^2}{2f^2}, \text{ and}$$

$$S_c(f) = 1/2 (S_{\Delta\theta}(f))$$

$S_c(f)$  is often expressed in dB relative to dBc/Hz and is related to the power spectrum observed on a spectrum analyzer. The National Institute of Standards and Technology ([www.nist.gov](http://www.nist.gov)) defines single-side band phase noise as the ratio of power in one phase modulation side-

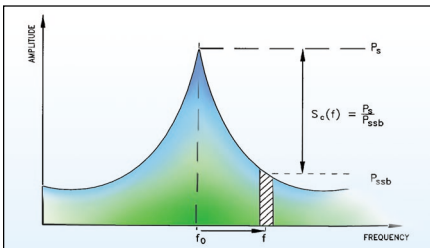


Figure 3. Single-sideband phase noise to carrier ratio

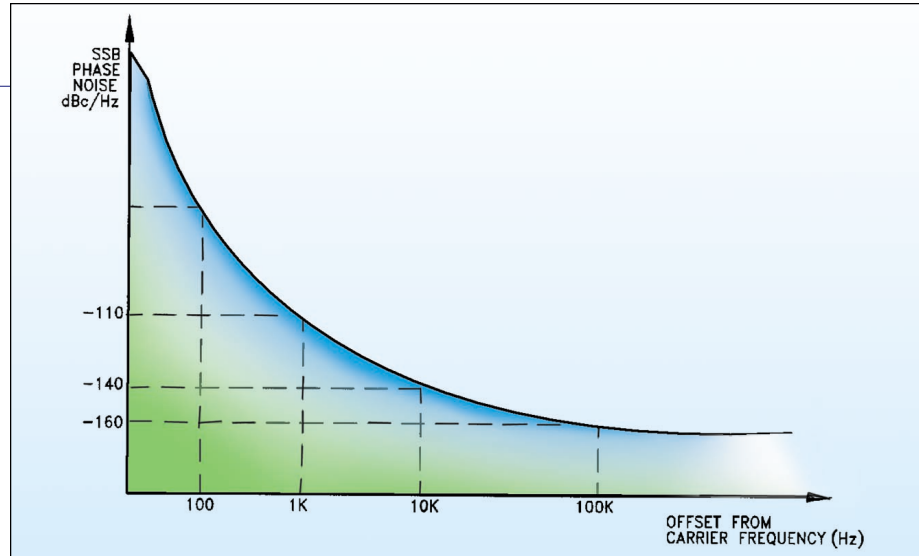


Figure 4. Single-sideband phase noise representation

band per Hertz bandwidth, at an offset  $f$  Hertz away from the carrier, to the total signal power. Here,  $f$  is the offset frequency from the carrier:

$$S_c(f) = \frac{P_s}{P_{ssb}}$$

where  $P_s$  is the carrier power and  $P_{ssbis}$  the sideband power in one Hz bandwidth at an offset frequency of  $f$  from the center. This is illustrated in figure 3.

The SSB phase noise is usually given logarithmically:

$$S_c(f) \text{ in dB} = 10 \log[S_c(f)]$$

This is shown in figure 4 as a spectral density plot of the phase modulation sidebands in the frequency domain. It is expressed in dBc/Hz.

The phase noise generated by a VCO is determined by the following:

- (a) Q-factor of the resonator.
- (b) Q of the varactor diode.
- (c) The active device used for the oscillating transistor.
- (d) Power supply noise.
- (e) External tuning voltage supply noise.

The noise contribution made by (d) and (e) can be minimized by careful choice of the power supplies. The phase noise of the VCO is, therefore, determined primarily by the overall Q of the circuit. To design a circuit with high Q, the tuning bandwidth must be made small, therefore a VCO designed for low phase noise performance will have a smaller tuning range.

### Ways to Minimize Noise

The following steps are recommended for obtaining the best overall performance from VCOs.

1. Power Supply ( $V_{cc}$ ) and tuning voltage ( $V_{tune}$ ) returns must be connected to

the printed circuit board's (PCB) ground plane. The VCO's ground plane must be the same as that of the PCB and, therefore, all VCO ground pins must be soldered direct to the PCB ground plane.

2. Adequate RF grounding is required. Chip decoupling capacitors must be inserted between the  $V_{cc}$  supply and ground.

3. High-quality, low noise power supplies must be used. Ideally, DC batteries — for both supply ( $V_{cc}$ ) and tuning ( $V_{tune}$ ) voltages — will provide the best overall performance.

4. Output must be correctly terminated with a good load impedance. It is also a good practice to use a resistive pad between the VCO and the external load.

5. Connections to the tuning port must be as short as possible and must be well screened, shielded, and decoupled to prevent the VCO from being modulated by external noise sources. A low noise power supply must be used for tuning voltage ( $V_{tune}$ ) supply.

**RF**

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